Modelling of coupled conduction and convection under dehumidifying conditions

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Abstract

The paper presents a generally applicable approach to the numerical simulation of coupled conduction and convection problems under dehumidifying conditions. Several examples are given which show good accuracy and reliability.

Keywords: Dehumidification; Coupled conduction-convection; Heat and mass transfer; Numerical simulation; Plate-fin exchangers; Tube-fin exchangers; Air-conditioning

1. Statement of the problem

The energy transport between a solid region V_S and a fluid region V_F can be modelled by the energy equation;

$$\rho c \frac{\partial t}{\partial \theta} + \rho c \boldsymbol{v} \cdot \nabla t = k \nabla^2 t \tag{1}$$

for the whole domain $V = V_S + V_F$. In Eq. (1) t is temperature, θ is time, ρ is density, c is the specific heat, $\boldsymbol{v}$ is the velocity vector and k is the thermal conductivity. The velocity vector $\boldsymbol{v}$ is identically equal to zero in the solid region, and thermophysical properties ρ, c and k assume different values in the solid and in the fluid regions. In the fluid regions reference is made to an incompressible, laminar flow of a constant property fluid. Consequently, ρ, c and k represent density, specific heat and thermal conductivity of the moist air mixture defined as a binary mixture of dry air and water vapour (see, for example, [1], chapter 7). For an incompressible, laminar flow of a constant property fluid the continuity, the Navier–Stokes and the species conservation equations in the domain V_F can be written in the form [2]:

$$\nabla \cdot \boldsymbol{v} = 0 \tag{2}$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial \theta} + \rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} = \mu \nabla^2 \boldsymbol{v} - \nabla p \tag{3}$$

$$\rho \frac{\partial \omega}{\partial \theta} + \rho \boldsymbol{v} \cdot \nabla \omega = \rho \mathcal{D} \nabla^2 \omega \tag{4}$$

respectively. In Eqs (3) and (4), μ is the dynamic viscosity, p is the deviation from the hydrostatic pressure, ω is the mass fraction of water vapour and $\mathcal{D}$ is the binary diffusion coefficient for the mixture of dry air and water vapour.

At external flow boundaries appropriate boundary conditions are, for example, at inflow, prescribed velocities $\boldsymbol{v} = \boldsymbol{v}_p$, temperatures $t = t_p$ and mass fractions $\omega = \omega_p$ and, at outflow, fully developed conditions $\partial \boldsymbol{v}/\partial n = \partial t/\partial n = \partial \omega/\partial n = 0$. Different boundary conditions, such as those on symmetry boundaries, can be modelled by combining prescribed values and normal derivatives. At external solid boundaries we only need appropriate conditions for the temperature field. At interfaces between solid and fluid regions we employ the no-slip condition $\boldsymbol{v} = \boldsymbol{v}_p = 0$ for the velocity field. To model the coupling between the energy equation (1) and the species conservation Eq. (4), we use first the ideal gas relationship to compute the value of the mass fraction of water vapour corresponding to the saturation pressure p_{vs} at the absolute wall temperature T_w:

$$\omega_s = \frac{p_{vs}(T_w)}{\rho R_v T_w} \tag{5}$$

Then, on the assumption that the condensate is promptly removed from the wall, we impose the condition:

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$$\omega = \omega_p = \omega_s \quad (6)$$

if condensation is going to start ($\omega_s < \omega_w$ and $\dot{m}''_v = 0$), or if condensation is already taking place ($\dot{m}''_v > 0$). On surfaces where no condensation occurs, we impose the boundary condition:

$$\frac{\partial \omega}{\partial n} = 0 \quad (7)$$

In the above equations ω_w is the mass fraction of water vapour at the wall, while $\dot{m}''_v$ is the specific mass flow rate of condensing water vapor ($\dot{m}''_v > 0$), or of evaporating water ($\dot{m}''_v < 0$). With respect to the energy Eq. (1), interfaces between solid and fluid regions are internal boundaries. Thus Eq. (1) ensures the continuity of temperature, and we only have to account for the additional latent heat flux:

$$q''_\lambda = \dot{m}''_v H_{vl} \quad (8)$$

on the surfaces where condensation takes place. In Eq. (8), H_{vl} is the enthalpy difference between water vapour at the temperature of the moist air and liquid water at the temperature of the solid surface. In the framework of the finite element method, $\dot{m}''_v$ can be evaluated from the 'nodal reactions' $(\dot{m}_v)_i$ on the interface nodes where ω is imposed in the discretised version of Eq. (4). During the assembly process we can thus add directly the nodal contributions:

$$(q_\lambda)_i = (\dot{m}_v)_i H_{vl} \quad (9)$$

to the right hand-side of the discretised version of Eq. (1).

2. Results and discussion

The Navier–Stokes equations are solved first by the finite element algorithm described in [3]. Once the velocity field has been found, the energy and the species conservation equation are solved before moving to the next step. Steady-state solutions are obtained from pseudo-transient simulations using a fully implicit scheme. In the space-discretisation, grids of eight-node linear elements are employed, with finer grid spacing near the walls and the inflow/outflow sections. To validate the model, we consider first a moving stream of moist air in contact with a cold plate of length $L = 0.1$ m, at a temperature below the dew point. The assumed boundary conditions are: $t_\infty = 30\,^\circ$C and relative humidity equal to 60% for the stream, yielding a mass fraction $\omega_\infty = 0.01587$; $t_w = 10\,^\circ$C and $\omega_w = \omega_s(10\,^\circ\text{C}) = 0.00765$ for the wall. The sensible heat flux q''_k and the latent heat flux q''_λ, computed from the steady-state reactions on the interface nodes in the energy and species conservation equation, behave as illustrated in Fig. 1. Furthermore the ratios between these fluxes are uniform and practically independent of the Reynolds number (Re). In fact we have $(q''_k/q''_\lambda) = 0.9013$ at $\text{Re}_L = 6700$ and $(q''_k/q''_\lambda) = 0.8964$ at $\text{Re}_L = 67000$. With reference to the heat convection coefficient α and the mass convection coefficient α_m, from the heat and mass transfer analogy

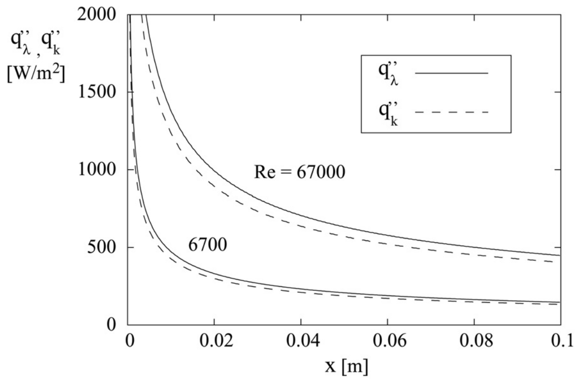

Fig. 1. Sensible and latent heat fluxes on a cold plate in contact with a moving stream of moist air.

$$\frac{q''_k}{q''_\lambda} = \frac{\alpha(t_\infty - t_w)}{\rho\,\alpha_m(\omega_\infty - \omega_w)H_{vl}} = \text{Le}^{2/3}\frac{c_p(t_\infty - t_w)}{(\omega_\infty - \omega_w)H_{vl}} \quad (10)$$

we obtain the analytical estimate $(q''_k/q''_\lambda) = 0.8959$ for $c_p = 1.029$ kJ/(kg K), $H_{vl} = 2478$ kJ/kg and a value of the Lewis number: $\text{Le} = k/(\rho c_p \mathcal{D}) = 0.835$.

In the second example we compare numerical and experimental results. In the experiment considered moisture condensation took place on a rectangular plane fin in a plate-fin exchanger [4]. The aluminum alloy 6061 was chosen as the base and fin material and the fins were 100 mm deep, 100 mm wide, and 2 mm thick. The test conditions were: frontal velocity of the moist air stream equal to 0.75 m/s; fin spacing equal to 3 mm; inlet dry bulb temperature equal to 27 °C; inlet relative humidity equal to 50%; base fin temperature equal to 9 °C. Using these values as boundary conditions, we computed velocity, temperature and concentration fields at steady state. The resulting distributions of temperature and specific mass flow rates of condensing vapor are illustrated in Fig. 2 (not to scale in transverse direction). It is worth noting that the boundary separating the dry and wet portions of the fin appears to be practically the same in Fig. 2 and in the corresponding visualization of Ref. [4].

The final example concerns a two-row tube-fin exchanger characterised by the following geometrical parameters: tube diameter $D = 10$ mm, channel height

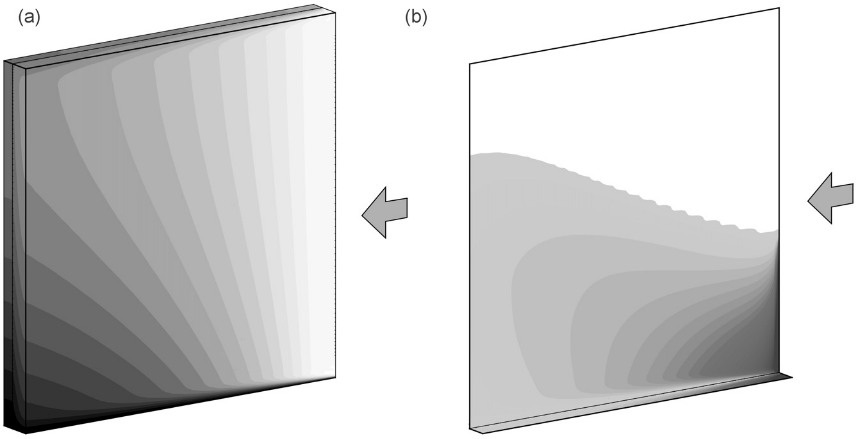

Fig. 2. Moisture condensation on a rectangular plane fin: (a) temperature field and (b) specific mass flow rates of condensing vapour.

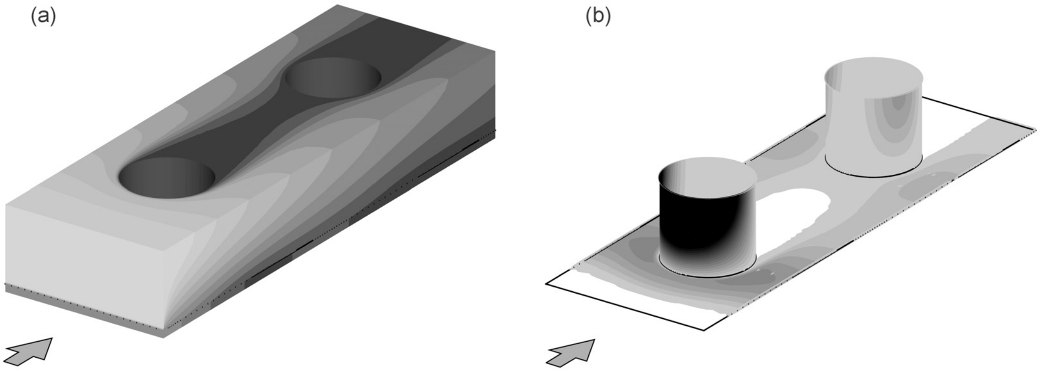

Fig. 3. Moisture condensation in a two-row tube-fin exchanger with an in-line arrangement of tubes: (a) temperature field and (b) specific mass flow rates of condensing vapour.

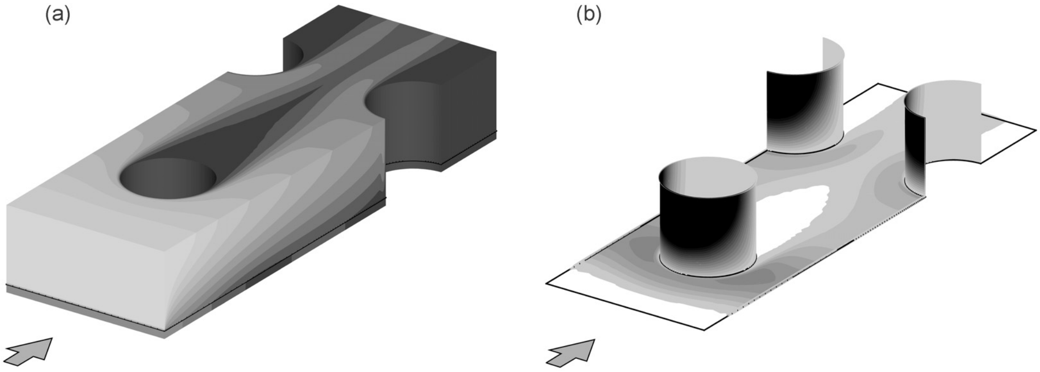

Fig. 4. Moisture condensation in a two-row tube-fin exchanger with a staggered arrangement of tubes: (a) temperature field and (b) specific mass flow rates of condensing vapour.

equal to 2 mm, fin thickness equal to 0.2 mm, transverse pitch equal to 20 mm and longitudinal pitch equal to 60 mm. The boundary conditions are: inlet dry bulb temperature equal to 30 °C, inlet relative humidity equal to 50% and tube temperature equal to 10 °C. For a Reynolds number $Re_D = 1000$, the resulting distributions of temperature and specific mass flow rates of condensing vapour are the ones illustrated in Fig. 3 for an in-line arrangement of tubes, and in Fig. 4 for a staggered arrangement of tubes. In this instance, figures are not to scale in the vertical direction.

3. Conclusions

Coupled conduction and convection, under dehumidifying conditions, can be modelled accurately and efficiently following the approach presented in this paper. The capability of dealing with complex geometries and realistic boundary conditions opens the door to new design procedures for compact heat exchangers used in air-conditioning and energy recovering applications.

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