A two-dimensional elastic model of pavements with thermal failure discontinuities

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Abstract

Elastic fields of a pavement resting on a granular base are studied considering the presence of existing thermal failure discontinuities (thermal cracks) typically observed in cold climates. The analysis endeavors towards the accurate prediction of crack spacing in asphalt pavements. A two-dimensional theoretical solution is derived and validated by comparison to numerical simulations. A simple method for obtaining an approximation of crack spacing is presented, which involves comparison of elastic fields to material tensile strength. The shear stress distribution can be used to predict the propensity towards debonding along the interface of the pavement surface and an underlying granular layer. An extension of this work to rigorously consider crack initiation and crack propagation using a cohesive zone fracture model and a viscoelastic constitutive model for the bulk material is underway.

Keywords: Elastic fields; Pavements; Discontinuity spacing; Thermal stress; Low temperature cracking; Tensile strength

1. Introduction

When an asphalt pavement is subjected to a thermal loading due to the ambient temperature change, thermal cracking can form across the width of the pavement [1]. Thermal cracking is one of the most devastating distresses that can occur in asphalt pavements in cold climates. Various empirical and mechanistic-empirical models [2] have been proposed to predict crack amount or crack spacing in a pavement. However, the thermal stress distribution in pavements has not been directly analyzed in those models though it is the dominant factor controlling thermal crack development.

To investigate the elastic fields of pavements, numerical methods [3,4] have been used to calculate the local stress and strain. Because the mechanical response of pavements depends upon material properties, interface properties, and geometry, while the quality of numerical simulations depends heavily on the quality of meshing, discretization aspects, etc., it is difficult to reliably obtain elastic fields for general cases from limited available numerical simulation results. Thus, closed-form analytical solutions are a valuable tool for researchers for model verification and, ultimately, to gain a better understanding of mechanical responses and damage mechanisms in pavements and similar structures [5].

Shen et al. [6] and Timm et al. [7], respectively, developed a one-dimensional (1D) model to predict tensile stress distribution in a pavement with frictional constraint. However, because the friction forces are driven from the bottom of the pavement and temperature distribution is not uniform in the thickness direction, the thermal stress will significantly change along the thickness of the pavement and a considerable shear stress will be induced along the bottom of pavement. Obviously, a 1D model can solve neither the thermal stress distribution in the thickness direction nor the shear stress distribution in the pavement, so a two-dimensional (2D) model is necessary to better analyze the thermal stress distribution.

In this paper, we first solve the general solution of displacement field for a 2D pavement subjected to a negative temperature change and having a linear temperature gradient as a function of depth. Considering the frictional boundary condition and thermal failure discontinuities, we explicitly obtain the displacement field in the pavement. Comparison with numerical

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Computational Fluid and Solid Mechanics 2005
K.J. Bathe (Editor)

simulations obtained by the finite element method (FEM) is used to verify the integrity of the proposed model. By considering the strength of the pavement materials, a procedure for estimating the thermal discontinuity spacing is then presented. Using the shear stress solution, a method for evaluating the propensity for debonding along the interface between the pavement and base layer in the vicinity of discontinuities is presented. It is noted that all material properties are taken as linear elastic for the current work. A discussion of model limitations and extensions is presented in Section 4.

2. Formulation

Consider an infinitely long pavement (width w, thickness h, Young's modulus E, Poisson's ratio ν, thermal expansion coefficient α) resting upon a granular base as illustrated in Fig. 1. With a change in ambient temperature, the top and bottom of the pavement will undergo different temperature changes, denoted as T_1 and T_2 ($T_1 < T_2$), respectively. Uniformly spaced discontinuities (thermal cracks) separated by a uniform distance of 2λ across the width of pavement are modeled. Here we set up the coordinates with the origin at the center between two discontinuities as seen in Fig. 1.

The pavement is assumed to be homogeneous material. An approximation of the steady state temperature distribution can be easily obtained as:

$$T(y) = T_2 + (T_1 - T_2)y/h \tag{1}$$

As the pavement is assumed to be bonded with the base layer, the cohesive force keeps the bottom of the pavement still in the plane. Because the thickness of the pavement is much smaller than its length and the top surface is free, generally the top surface of pavement remains approximately flat during the temperature change if no debonding happens along the interface between the pavement and the base. Thus, we assume that all points of a plane normal to the y direction is still in the same plane after deformation, i.e.

$$u_y(x,y) = u_y(y) \tag{2}$$

Because the upper surface is free, the thermal strain in the y direction is not constrained. Thus, we assume that the stress in the y direction is zero, i.e.

$$\sigma_y(x,y) = 0 \tag{3}$$

For this 2D elastic problem, the constitutive law is:

$$\sigma_x = E(\varepsilon_x - \alpha T), \quad \tau_{xy} = \mu\gamma_{xy} \tag{4}$$

Considering the equilibrium condition in x direction, we can write:

$$Eu_{x,xx} + \mu u_{x,yy} = 0 \tag{5}$$

Using the method of separation of variables, we can find the general solution as:

$$u_x(x,y) = (A_1 e^{cx} + A_2 e^{-cx})[B_1 \sin(dy) + B_2 \cos(dy)] \tag{6}$$

where A_1, A_2, B_1 and B_2 are constants to be decided by the boundary conditions and $d = \sqrt{E/\mu}c$.

From the symmetry of the geometry and the free upper surface, we write:

$$u_x(0,y) = 0; \quad u_{x,y}(x,h) = 0 \tag{7}$$

Using the above boundary conditions, we simplify Eq. (6) as follows:

$$u_x(x,y) = B(e^{cx} - e^{-cx}) \cos d(h - y) \tag{8}$$

Along the bottom of the pavement, the interfacial friction force may provide a resistance to the displacement in the x direction such that:

$$\tau_{xy}(x,0) = ku_x(x,0) \tag{9}$$

where k is the friction coefficient for unit thickness. Substituting Eq. (8) into Eq. (9), we obtain:

$$d = \frac{k}{\mu \tan(dh)}; \quad c = \sqrt{\mu/E}d \tag{10}$$

It is noted that d is solved numerically by a recursive method.

Along the surface of the discontinuity, the stress σ_x should be zero. Due to the assumptions implied by Eqs.

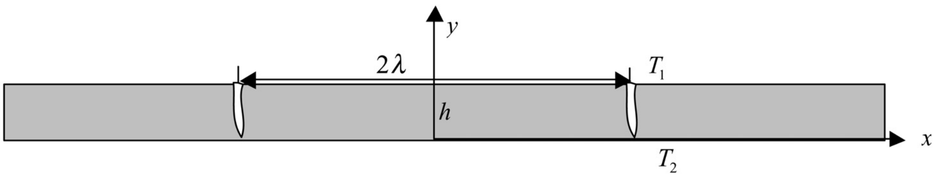

Fig. 1. A 2D pavement with discontinuities due to thermal loading.

(2) and (3), this boundary condition cannot be rigorously satisfied at every point. Here we set the total normal force as zero, namely:

$$\int_{y=0}^{h} \sigma_x(\lambda,y)dy = 0 \tag{11}$$

Substituting Eq. (1) into Eq. (4) and Eq. (4) into Eq. (11) yields:

$$B = \alpha dh \frac{T_1 + T_2}{2c\sin(dh)(e^{c\lambda} + e^{-c\lambda})} \tag{12}$$

Thus, we obtain the explicit solution in Eq. (8) with constants d and in c in Eq. (10), and B in Eq. (12).

3. Results and discussion

To verify the integrity of the proposed analytical model, comparisons are made with an FEM simulation by the software DIANA (DIsplacement ANAlyzer). Here we use $E = 14.0\,\mathrm{GPa}$; $v = 0.2$; $\alpha = 1.8{*}10^{-5}1/\mathrm{K}$; $T_1 = -30\,\mathrm{K}$; $T_2 = -25\,\mathrm{K}$; $h = 0.2\,\mathrm{m}$; $\lambda = 4\,\mathrm{m}$. We draw the displacement distributions in the x direction at the top and the bottom of the pavement as seen in Fig. 2. The model developed by Timm et al. [7] is also shown in this figure but it can only provide identical prediction for both the top and the bottom of the pavement because it is a 1D model. Figure 2 shows that the proposed theoretical prediction of the displacement is very close to FEM simulation whereas in the neighborhood of the discontinuity the FEM simulation provides a slightly higher estimate at the bottom and a lower estimate at the top. The 1D prediction provides a smaller prediction when x is small. In the neighborhood of the discontinuity, the prediction is between those at the top and bottom for either the FEM simulation or 2D theoretical prediction.

Figure 3 shows the comparisons of stress distributions in the x direction at the top and the bottom of the pavement. Because three methods provide very close predictions in the range of $0 \leq x \leq 2.0\,\mathrm{m}$, as seen in Fig. 2, we only show the range of $2.0\,\mathrm{m} \leq x \leq 4.0\,\mathrm{m}$. In Fig. 3(a) we can see on the top surface the proposed 2D model provides a good agreement with the FEM simulation for tensile stress except at the neighborhood of the discontinuity as expected, whereas the 1D prediction is not as close to the FEM simulation. Figure 3(b) illustrates tensile stress and shear stress distributions along the bottom of the pavement. In the neighborhood of the discontinuity the FEM simulation presents a large change with respect to the proposed model solution due to the singular effect. In the other range, the proposed 2D model fits the FEM simulation well for both tensile stress and shear stress. However, the 1D prediction only provides tensile stress, which is lower than the 2D prediction and the FEM simulation.

In Fig. 3 we can find that the maximum tensile stress is at the midpoint on the top surface and the maximum shear stress is at the bottom of the discontinuities, and the maximum tensile stress is higher than the maximum shear stress. Substituting Eq. (8) into Eq. (4) we obtain:

$$\sigma_x^{\max} = E\left(\alpha dh \frac{T_1 + T_2}{\sin(dh)(e^{c\lambda} + e^{-c\lambda})} - \alpha T_1\right);$$
$$\tau_{xy}^{\max} = \mu\alpha d^2 h \frac{T_1 + T_2}{2c}\tanh(c\lambda) \tag{13}$$

where the former is positive and the latter is negative at low temperature. With the decrease of the ambient temperature, the maximum tensile stress increases. When it reaches the tensile strength of the pavement material, S, a new discontinuity would be initiated from

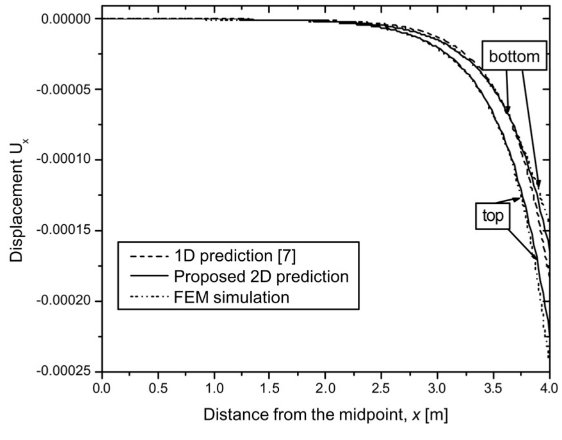

Fig. 2. Displacement distributions along the top and bottom surface of pavement.

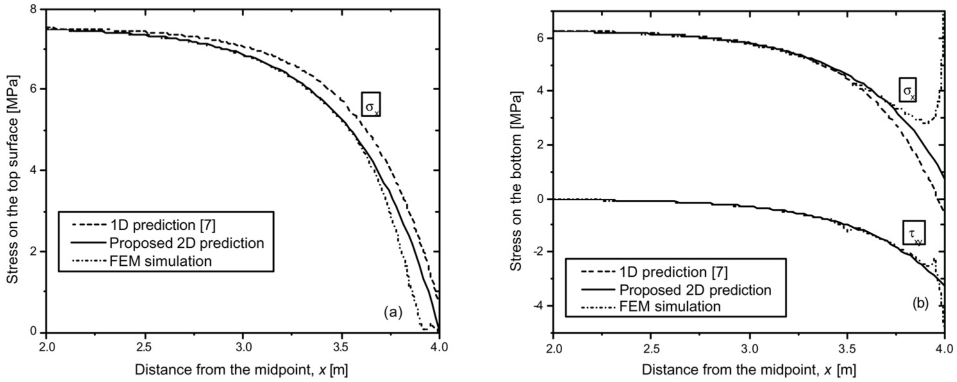

Fig. 3. Stress distributions along (a) the top and (b) bottom surface of pavement.

the midpoint on the top surface. Then the maximum tensile stress will move to the midpoint of the new interval, at a magnetude that is much lower than the tensile strength. Thus given the geometry, material properties, and temperature distribution of a pavement, we can solve the critical discontinuity spacing λ^c, in which the maximum tensile stress is equal to the tensile strength, i.e. $\sigma_x^{\max}(\lambda^c) = S$. Thus, we can calculate the critical discontinuity spacing λ^c [5,7]. Although the maximum shear stress is not as considerable as the tensile stress, when the interface between the pavement and the granular base is not strong, the shear stress may induce the debonding of the interface starting at the bottom of the discontinuities, which will cause curling of the pavement.

4. Limitations and extensions

It is noted that the pavement material is assumed to be linear elastic for the current work. Although this may be a reasonable approximation at very low temperatures [7], a rigorous consideration of crack initiation and crack propagation and a viscoelastic constitutive model for the bulk material is ultimately needed. In addition, pavement temperature gradients may also be nonlinear. Nevertheless, the current elastic analysis is an important first step in this direction, as it establishes a rigorous baseline that can be used in the development of pavement simulations with time-dependent and/or non-linear material properties and interface conditions. Future modeling efforts are planned to compare this model with field data from Mn/ROAD facilities and laboratory data.

Acknowledgments/disclaimer

This work is sponsored by the Federal Highway Administration National Pooled Fund Study 776, whose support is gratefully acknowledged. The results and opinions presented herein are those of the authors and do not necessarily reflect those of the sponsoring agency. We would also thank Mr. S.H. Song and Mr. M.P. Wagoner for their valuable discussions.

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