Study of interaction curves for composite laminate with cutout

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Abstract

The presence of a cutout in a laminated composite plate is of interest to the analysts. The main objective here is to study the interaction curves for a laminated composite plate with a centrally located cutout subjected to in-plane and shear loading, using a simple higher-order shear deformation theory based on four displacement functions (u_0, v_0, w_b, w_s) instead of the five that are commonly used in most higher-order theories. The finite element method is employed to study the buckling of laminated plates.

Keywords: Cutout; Interaction curves; Laminates; Buckling

1. Introduction

Composites offer unique opportunities in design. Composite plates with cutouts are extensively used in many structures because of their high stiffness–to–weight and strength–to–weight ratios. Cutouts change the mechanical behavior of plates. Hence the stability analysis of plates with cutouts is of technical importance for understanding the behavior of systems under different types of loading. Various plate theories have been developed over the years to understand the mechanical behavior of composite laminates. Some studies [1,2] have shown that the transverse shear effect is quite significant in layered composite plates due to the high ratio of elastic modulus to shear modulus, which makes classical laminate plate theory unsuitable for the analysis. To produce a better representation of the shear distribution across the thickness, Reddy [3] proposed a higher-order shear deformation theory with five unknowns. A simplified theory proposed by Lim et al. [4] involves only four unknowns instead of the five for Reddy [3]. This simplified theory allows using C^1 continuous plate bending element, which is free from any shear locking effect. The stability of plates with perforated holes is studied by Yettram and Brown [5]. They employ conjugate/load displacement method to analyze the structural stability of perforated plates. Prabhakara and Dutta [6] studied the vibration and buckling behavior of plates with centrally located cutouts. In this investigation the interaction curves for composite plates with a centrally located cutout have been studied.

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2. Formulation for buckling load

A plate theory proposed by Lim et al. [4] for isotropic plates is discussed and extended to a laminated composite plate. The displacement field includes classical plate theory and first-order shear deformation theory as its subset and accounts for the parabolic variation of transverse shear strains, which also accounts for the surface boundary condition of zero transverse shear stress (and hence shear strains) at the top and bottom surfaces of the plate.

The displacement field $V(x, y, z)$ is expressed as

$$\{V(x,y,z) = [u(x,y,z), v(x,y,z), w(x,y,z)]\}^{\mathrm{T}} \tag{1}$$

where $u(x, y, z)$, $v(x, y, z)$ and $w(x, y, z)$ are given by

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z w_{\mathrm{b},x}(x,y) + z^2 \varphi_x(x,y) + z^3 \Psi_x(x,y) \\ v(x,y,z) &= v_0(x,y) - z w_{\mathrm{b},y}(x,y) + z^2 \varphi_y(x,y) + z^3 \Psi_y(x,y) \\ w(x,y,z) &= w_{\mathrm{b}}(x,y) + w_{\mathrm{s}}(x,y) \end{aligned} \tag{2}$$

where u_0 and v_0 are the mid-plane displacements along the x and y directions.

The transverse displacement component w_b is such that its derivatives are numerically equal to the rotation of the cross section (i.e. $\varphi = -\nabla w_{\mathrm{b}}$) and w_{s} is the displacement due to the effect of transverse shear deformation of the cross section.

The linear strain displacement relationships for the higher-order theory under consideration can be written as follows.

$$
\begin{aligned}
\varepsilon_{xx} &= u_{0,x} - zw_{\mathrm{b},xx} + z^2\varphi_{x,x} + z^3\Psi_{x,x} \\
\varepsilon_{yy} &= v_{0,y} - zw_{\mathrm{b},yy} + z^2\varphi_{y,y} + z^3\Psi_{y,y} \\
\varepsilon_{zz} &= 0 \\
\gamma_{xy} &= u_{0,y} + v_{0,x} - 2zw_{\mathrm{b},xy} + z^2(\varphi_{y,x} + \varphi_{x,y}) + \\
&\quad z^3(\Psi_{y,x} + \Psi_{x,y}) \\
\gamma_{xz} &= w_{\mathrm{s},x} + 2z\varphi_x + 3z^2\Psi_x \\
\gamma_{yz} &= w_{\mathrm{s},y} + 2z\varphi_y + 3z^2\Psi_y
\end{aligned} \tag{3}
$$

where a comma (,) denotes partial derivatives.

The surface boundary conditions that the transverse shear stresses vanish at the top and bottom faces are equivalent to the requirement that the corresponding strains be zero on these surfaces. Hence the resulting displacement field, after satisfying these conditions, is given by

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) - zw_{\mathrm{b},x}(x,y) - \frac{4z^3}{3h^2}w_{\mathrm{s},x}(x,y) \\
v(x,y,z) &= v_0(x,y) - zw_{\mathrm{b},y}(x,y) - \frac{4z^3}{3h^2}w_{\mathrm{s},y}(x,y) \\
w(x,y,z) &= w_{\mathrm{b}}(x,y) + w_{\mathrm{s}}(x,y)
\end{aligned} \tag{4}
$$

2.1. Finite element model

For the finite element discretization of the plate, rectangular four-noded elements are used along with a linear Lagrange interpolation function to model the geometry of the plate. A C^1 continuous shear flexible element based on the higher-order theory is developed using the Hermite interpolation functions. The components of displacements can be expressed in terms of the four unknowns, which can be written as

$$\{\delta\}^{\mathrm{T}} = \{u_0, v_0, w_{\mathrm{b}}, w_{\mathrm{s}}\} \tag{5}$$

2.2. The pre-buckled problem

When the reference configuration is stress free, the pre-buckled problem is solved for the linear elasticity solution by minimizing the total potential energy in the standard way to get

$$[\bar{K}]\,\{\bar{\delta}\} = \{F\} \tag{6}$$

for a reference load T_{i}^0. The buckled state is obtained as a perturbation of this pre-buckled state. The pre-buckled configuration is not stress free and hence the work done by the nonlinear strain terms may not be negligible. Therefore the perturbational total potential energy expression is written as

$$\Pi(u_{\mathrm{i}}) = \frac{1}{2}\int C_{ijkl}\varepsilon_{ij}\varepsilon_{kl}dv + \frac{1}{2}\int \sigma_{ij}^0 u_{\alpha,i}u_{\alpha,j}dv - \int T_i u_i dA \tag{7}$$

The second integral in Eq. (7) represents the work done by the pre-buckled stresses against the nonlinear strain terms. The work done by σ_{ij}^0 due to the linear part of strain is cancelled by the work done by T_i^0. In the structural stability problems, $T_i = 0$, and the objective is to find the lowest scalar multiplier, λ, of σ_{ij}^0 and the corresponding nontrivial displacement function u_i such that

$$\Pi(u_{\mathrm{i}}) = \frac{1}{2}\int C_{ijkl}\varepsilon_{ij}\varepsilon_{kl}dv + \frac{\lambda}{2}\int \sigma_{ij}^0 u_{\alpha,i}u_{\alpha,j}dv \tag{8}$$

is minimum. The stress field $\sigma^0{}_{ij}$ is called the 'pre-buckling stress state', and the critical load (also called the 'bifurcation-buckling load') is $\lambda_{\min}T_i^0$.

3. Numerical results and discussion

In the present investigation, the pre-buckled problem is solved first. The pre-buckled stress is computed and this stress is used in the calculation of buckling load. This is required since the stresses in the domain vary due to presence of cutouts.

3.1. Boundary conditions

The following types of geometric boundary conditions are considered in the analysis:

- $u = 0$ and $v \neq 0$ along loaded edges;
- $u \neq 0$ and $v = 0$ along unloaded edges;
- simply-supported (S): $w_b = w_s = 0$;
- clamped (C): $w_b = w_s = w_{b,n} = w_{s,n} = 0$;

where, $n = x$ or y depending on the side of the plate.

3.2. Convergence study

The convergence study of a plate with a cutout is made by considering two kinds of meshes, one being the graded mesh and the other the equal mesh. It is found that a 9 × 9 graded mesh gives a reasonably good result when compared to a uniform mesh. This is because the graded mesh efficiently captures the stresses near the corners of the cutout.

3.3. Validation of plate model with cutout

The results for an isotropic plate with a cutout are compared with the results given in the [5]. Figure 1 shows the graph of buckling load for an isotropic plate with varying hole–to–plate ratio, with all the edges simply supported. A square plate with a square hole is considered here. From the figure it can be observed that the buckling load decreases continuously with the increase in hole size. For both models, i.e. the present analysis and the one given in [5], the results are very close for smaller hole sizes. Discrepancy is observed for larger hole sizes.

3.4. Interaction curves

This section deals with the study of interaction curves for laminated composite plates under biaxial loading. The material properties employed for this study are taken from [7]. Figures 2 and 3 show the interaction curves for a plate with a centrally located cutout for plate of aspect ratio 2. Kx and Ky represent the buckling parameters. Figure 2 shows the interaction curves for a thick plate. From this figure it can be observed that there is a sudden change in slope associated with the interaction curves. The interaction curves for $\theta = 45^\circ$ and $\theta = 60^\circ$ laminate intersect at one particular point. At this point it can be said that these two laminations bear the same amount of buckling load. The curves are almost perpendicular to the y-axis for one portion, and almost perpendicular to the x-axis for the other portions. This is due to the fact that at these portions of the curves for a small decrease in the value of N_x the strength of the laminate to carry N_y increases rapidly. Figure 3 shows the interaction curve for a thin plate. It can be observed from the figure that the $\theta = 45^\circ$ and $\theta = 60^\circ$ laminates

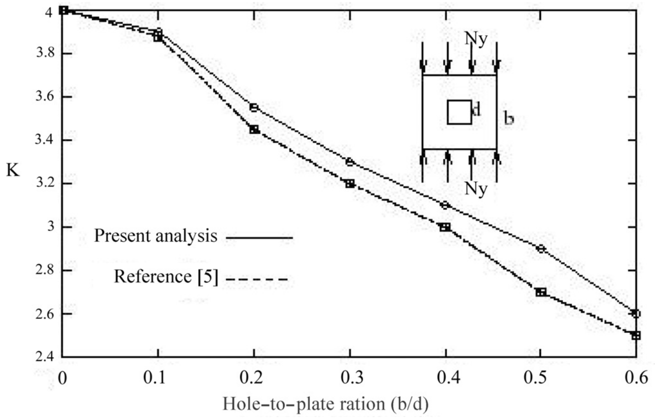

Fig. 1. Non-dimensionalized buckling load versus hole–to–plate ratio.

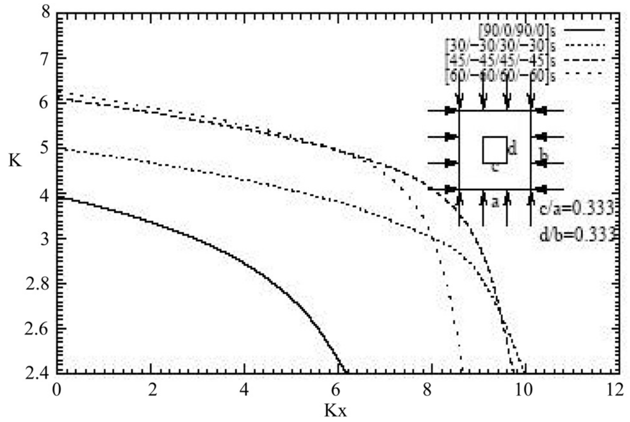

Fig. 2. Interaction curves for biaxial in-plane compressive loads N_x and N_y with $a/b = 2$ and $a/h = 10$.

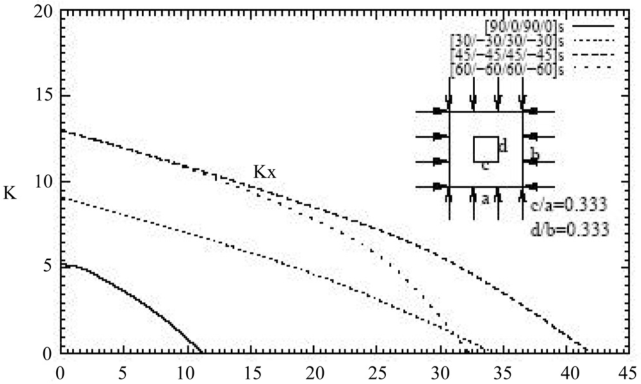

Fig. 3. Interaction curves for biaxial in-plane compressive loads N_x and N_y with $a/b = 2$ and $a/h = 100$.

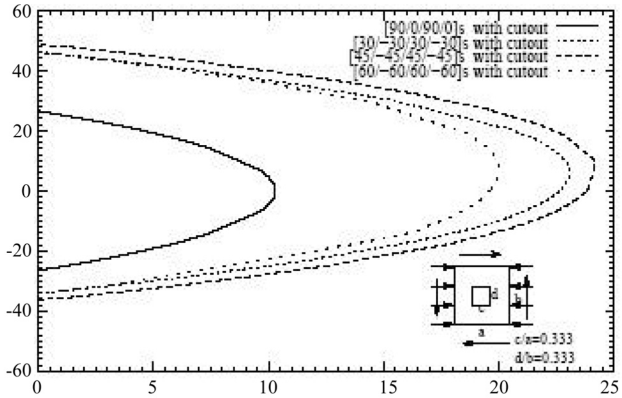

Fig. 4. Comparative study of stability envelope for biaxial in-plane compressive and shear loads N_x and N_{xy} with $a/b = 1$ and $a/h = 10$.

give the maximum value of $N_x + N_y$ and also that they bear the same amount of buckling load for a certain portion of the curve. For a cross-ply laminate, the change in slope occurs at high value of N_y, whereas for the $\theta = 45^\circ$ laminate the change in slope occurs at high values of N_x. Figure 4 shows the interaction curves for a thick composite plate with a centrally located cutout under in-plane compressive and shear loading. The interaction curves are symmetric about the x-axis for a cross-ply laminate, whereas for other laminates these are no longer symmetric. From the figures, it can be observed that the value of $N_x + N_{xy}$ is more for positive shear as compared to negative shear. Also there is no such point at which two or more curves correspond to the same amount of buckling load.

4. Conclusions

1. The buckling load of a laminated plate decreases in the presence of a cutout.
2. The interaction curves for N_x and N_y with cutout are very much different than those for a laminate without a cutout.
3. The interaction curves for biaxial loading are very much different for thin and thick plates.
4. For biaxial compressive and shear loads the interaction curve is symmetric for a cross-ply laminate and this behavior is not observed with other laminates.

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