Crack problems with a NGF/OQM BEM formulation for the scalar wave equation

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Abstract

This paper discusses the application of the numerical Green's function boundary element approach coupled with the operational quadrature method for the solution of dynamic crack problems governed by the scalar wave equation. Numerical examples are presented, allowing for the verification of the accuracy of the proposed procedure.

Keywords: Boundary element method; Numerical Green's function; Operational quadrature method; Fracture mechanics

1. Introduction

An efficient boundary element procedure [1] to deal with fracture mechanics problems is obtained with the implementation of the associated Green's function [2] acting as the fundamental solution. The advantage of this formulation is the formal elimination of integration of the unknown crack surface variables from the basic implementation. When the problem at hand is in the time-domain, an elegant way of solving is using the so-called operational quadrature method (OQM) [3,4,5], where the convolution integral, present in time-domain boundary element method (BEM) formulation, is substituted by a quadrature formula, whose weights are computed using the Laplace transform of the fundamental solution and a multistep method. The final solution to the problem, however, is still obtained in the time domain.

2. Numerical Green's function

The fundamental Green's function can be written in terms of the superposition of a full space fundamental solution plus a complementary part which provides satisfaction of the traction-free requirement over the crack surfaces. This Green's function can be represented as:

$$u^G(\xi,x) = u^t(\xi,x) + u^c(\xi,x) \tag{1}$$

$$p^G(\xi,x) = p^t(\xi,x) + p^c(\xi,x) \tag{2}$$

where u^G and p^G are the fundamental displacements and tractions; u^t and p^t define the standard full space solution and u^c and p^c are the complementary solution components, corresponding to a loaded crack problem, determined by:

$$u^c(\xi,x) = \int_{\Gamma^I} p^G(x,\zeta)c(\xi,\zeta)d\Gamma(\zeta) \tag{3}$$

$$p^c(\xi,x) = \int_{\Gamma^I} P^G(x,\zeta)c(\xi,\zeta)d\Gamma(\zeta) \tag{4}$$

where $c(\xi,\zeta) = u^c(\xi,\zeta^S) - u^c(\xi,\zeta^I)$ is the crack opening displacement of the Green's function and the superscripts S and I stand for 'superior' and 'inferior' surfaces of the crack $\left(\Gamma^F = \Gamma^S \cup \Gamma^I\right)$.

3. The operational quadrature method for crack problems

The time-domain BEM corresponding to a scalar wave problem is:

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$$4\pi C(\xi)u(\xi,t) = \int_\Gamma \int_0^{t^+} u^G(x,t;\xi,\tau)p(x,\tau)d\tau d\Gamma(x) - \int_\Gamma \int_0^{t^+} p^G(x,t;\xi,\tau)u(x,\tau)d\tau d\Gamma(x) \quad (5)$$

In the OQM, the convolution integrals can be approximated by a quadrature formula such as:

$$\int_0^{t^n} \int_\Gamma u^G(x,t;\xi,\tau)p(x,\tau)d\Gamma d\tau = \sum_{k=0}^{n} g^j_{n-k}(x,\xi,\Delta t)p^j_k(x) \quad n = 0,1\ldots,N \quad (6)$$

$$\int_0^{t^n} \int_\Gamma p^G(x,t;\xi,\tau)u(x,\tau)d\Gamma d\tau = \sum_{k=0}^{n} h^j_{n-k}(x,\xi,\Delta t)u^j_k(x) \quad n = 0,1\ldots,N \quad (7)$$

In this way, the quadrature weights are represented as follow:

$$g^j_n(x,\xi,\Delta t) = \frac{\rho^{-n}}{L}\sum_{l=0}^{L-1}\int_{\Gamma_j}\hat{u}^G\left(x,\xi,\frac{\gamma(\rho e^{il2\pi/L})}{\Delta t}\right)\phi^j(x)d\Gamma(x)e^{-inl2\pi/L} \quad (8)$$

$$h^j_n(x,\xi,\Delta t) = \frac{\rho^{-n}}{L}\sum_{l=0}^{L-1}\int_{\Gamma_j}\hat{p}^G\left(x,\xi,\frac{\gamma(\rho e^{il2\pi/L})}{\Delta t}\right)\phi^j(x)d\Gamma(x)e^{-inl2\pi/L} \quad (9)$$

where $\hat{u}^G$ and $\hat{p}^G$ are the fundamental displacement and tractions in Laplace domain.

4. Numerical examples

The first example represents a linear crack in an infinite medium with a unit time step load. The total crack length is 4, the time was divided into equal intervals Δt such that $\beta = c\Delta t/l = 0.3787$ (l is the element length) and $c = 1.0$ (wave velocity). The result is shown in Fig. 1 and is compared with a time-domain BEM formulation.

The second example is a one-dimensional rod under a Heaviside-type forcing function. The geometry is shown in Fig. 2, $2a = 4$, the boundary was discretized using 52 quadratic boundary elements. The results, as shown in Figs. 3 and 4, are compared with a time-domain BEM and a pure OQM BEM formulation.

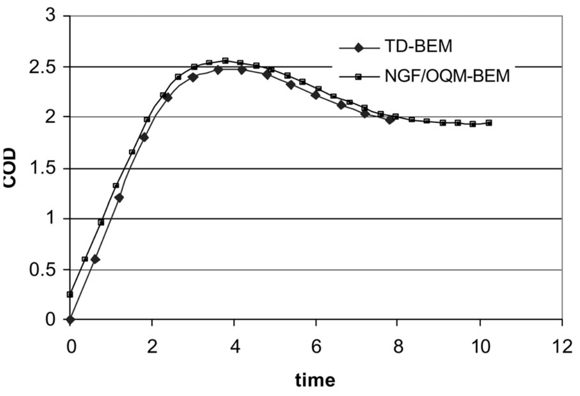

Fig. 1. COD vs. time.

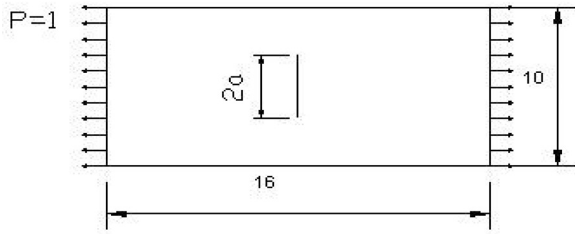

Fig. 2. Boundary conditions and geometry definitions.

5. Conclusion

The NGF procedure has been applied, together with the OQM, to solve dynamic problems involving the simulation of cracks in scalar field problems. The accuracy of the solutions obtained illustrates the accuracy of the combined approach.

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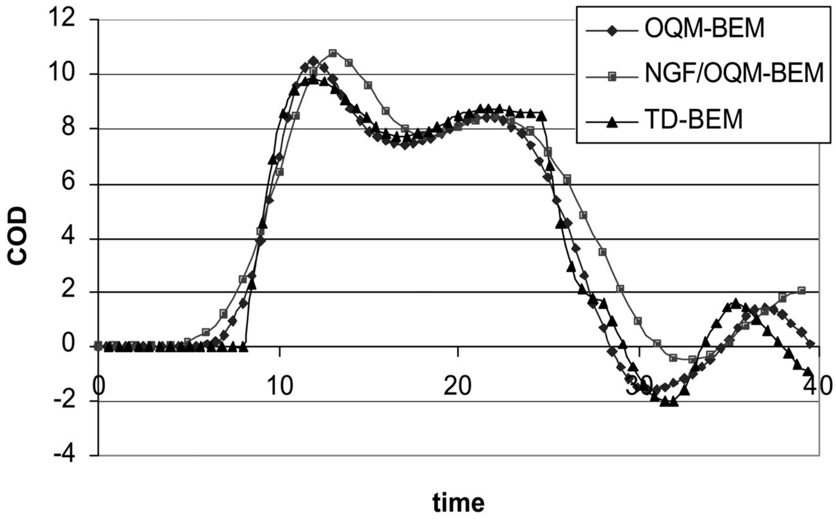

Fig. 3. COD vs. time.

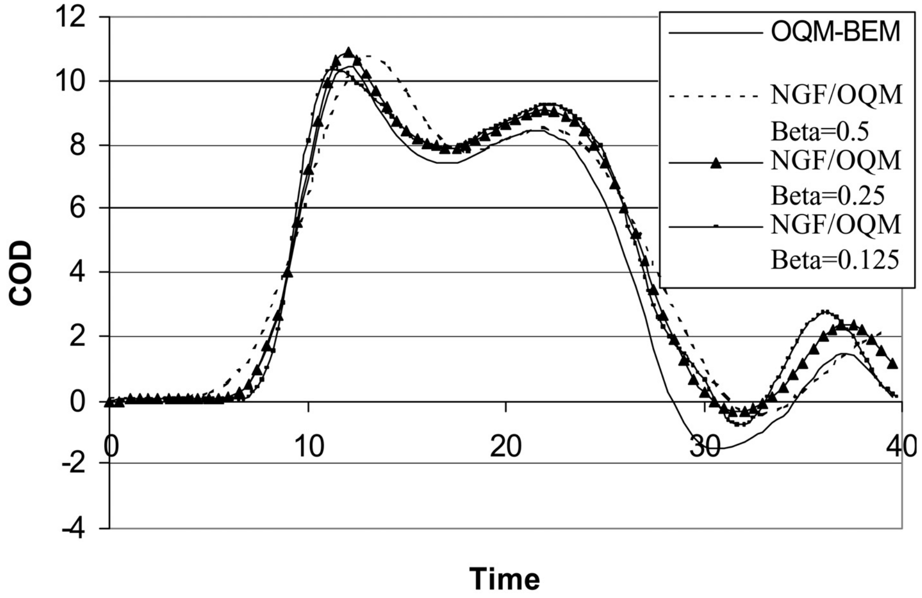

Fig. 4. COD for different values of β.

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