Dynamic bending and domain wall motion in piezoelectric laminated actuators under ac electric fields

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Abstract

We investigate the nonlinear electromechanical response of piezoelectric laminated actuators under alternating current (ac) electric fields both analytically and experimentally. A laminated beam theory solution is developed for the cantilever piezoelectric/metal/piezoelectric actuator, and the effects of ac electric fields on the deflection are analyzed. A simple phenomenological model of a vibrating domain wall in electric fields is used, and the macroscopic actuator response is predicted. A nonlinear three-dimensional finite element model is also developed. Bending tests are used to validate the predictions using bimorph-type bending actuators made with soft lead zirconate titanate (PZT) layers and a metal sheet. Theoretical predictions of the dynamic bending behavior are in excellent agreement with measured values.

Keywords: Elasticity; Finite element method; Material testing; Piezocomposite; Beam; Dynamic bending; Domain wall motion; Smart materials

1. Introduction

The adaptability properties of piezoelectric ceramics make them highly desirable materials for use in actuation. The typical piezoelectric actuators include multilayer stacked actuators, bimorph-type bending actuators, and composite actuators. When large displacement is required, bimorph-type actuators are usually used. In some actuator applications, fairly large electric fields are applied to piezoelectric materials at relatively low frequencies. Under large electric fields, material properties provided by manufacturers are no longer applicable to describe device performance since they were measured at a weak signal level [1]. Recently, Shindo et al. [2] studied performance of multilayered actuators in a wide electric field range, and found that the electroelastic field concentrations induce the polarization switching near the electrode tip and the strain vs. electric field curves show the nonlinear behavior. They also showed that the difference between calculated and measured strains near the electrode tip becomes larger at a higher electric field. Similar nonlinear behavior was observed in piezoelectric disks with circular electrodes [3].

As we know, the electromechanical properties in ferroelectric materials are caused not only by the ionic displacement (the intrinsic effect), but also by the movements of domain walls (the extrinsic contribution). Experimental studies on soft PZT have shown that the dielectric and piezoelectric coefficients increase with electric field due to the extrinsic contribution at room temperature [4,5]. The extrinsic effect is very complicated because it involves interactions of several length scales, i.e. the interactions between ions, domains, and even diffirent phases.

There remains a need at present for efficient numerical models and methods for predicting basic macroscopic material response while simultaneously accounting for microscale phenomena, such as domain switching and domain wall motion. This paper constitutes a continuing study of the previous work [6] on nonlinear displacement properties of piezoelectric laminated actuators, and presents theoretical and experimental results on the nonlinear bending behavior due to domain wall motion under ac electric fields.

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Computational Fluid and Solid Mechanics 2005
K.J. Bathe (Editor)

2. Electroelastic analysis

2.1. Basic equations and domain wall motion

The basic equations for piezoelectric ceramics are:

$$\sigma_{ji,j} = \rho u_{i,tt} \tag{1}$$

$$D_{i,i} = 0 \tag{2}$$

$$\varepsilon_{ij} = s_{ijkl}\sigma_{kl} + d_{kij}E_k \tag{3}$$

$$D_i = d_{ikl}\sigma_{kl} + \in_{ik} E_k \tag{4}$$

$$\varepsilon_{ij} = (u_{j,i} + u_{i,j})/2 \tag{5}$$

$$E_i = -\phi_{,i} \tag{6}$$

where σ_{ij}, D_i, ε_{kl} and E_k are the stresses, electric displacements, strains and electric fields, u_i and ϕ are the displacements and electric potential, respectively, ρ is the mass density, and a comma followed by an index denotes partial differentiation with respect to a space coordinate x_i (i = 1, 2, 3) or the time t. We have employed Cartesian tensor notation and the summation convention for repeated tensor indices. s_{ijkl}, d_{kij} and $\in_{ik}$ are the corresponding elastic compliances, and piezoelectric and dielectric constants, which satisfy the following symmetry relations:

$$s_{ijkl} = s_{jikl} = s_{ijlk} = s_{jilk} = s_{klij}, \quad d_{kij} = d_{kji}, \quad \in_{ij} = \in_{ji} \tag{7}$$

Arlt and Dederichs [7] have developed a phenomenological theory to calculate the contributions of domain wall motions for ferroelectric ceramics. For simplicity here, the direction of the applied ac electric field E_0 exp $(i\omega t)$ is parallel to the direction of spontaneous polarization P^s in one of the domains as shown in Fig. 1; ω is the input frequency. The induced strain $\Delta\varepsilon_{ij}$ and the change of the electric dipole moment ΔP_i of this basic unit due to the domain wall displacement Δl can be written as:

$$\Delta\varepsilon_{11} = -\frac{\Delta l}{l}\gamma^s, \quad \Delta\varepsilon_{22} = 0, \quad \Delta\varepsilon_{33} = \frac{\Delta l}{l}\gamma^s,$$
$$\Delta\varepsilon_{12} = 0, \quad \Delta\varepsilon_{23} = 0, \quad \Delta\varepsilon_{31} = 0 \tag{8}$$

$$\Delta P_1 = -\frac{\Delta l}{l}P^s, \quad \Delta P_2 = 0, \quad \Delta P_3 = -\frac{\Delta l}{l}P^s \tag{9}$$

where l is the domain width and γ^s is the spontaneous strain. The equation of domain wall motion may be written as [7–9]:

$$m_D\Delta l_{,tt} + \beta\Delta l_{,t} + f_D\Delta l = -\frac{\partial W}{\partial \Delta l}l \tag{10}$$

where m_D is the effective mass per unit area of wall, β is the damping constant of the wall motion, f_D represents the force constant for the domain wall motion process, and $W = -(\sigma_{ij}\Delta\varepsilon_{ij} + E_i\Delta D_i)/2$ is the induced energy.

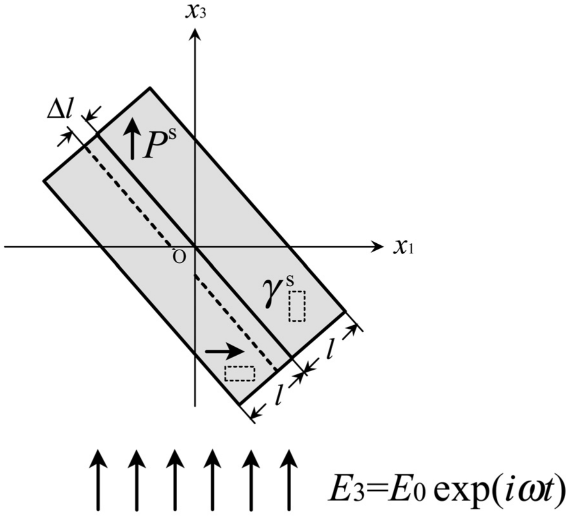

Fig. 1. Basic unit of a piezoelectric crystallite with a domain wall.

Damping may be occasioned by coupling with lattice vibrations and other causes, but for the present we set $\beta = 0$ purely for convenience [8].

In the following we will analyze only the domain wall movement under an ac electric field $E_3 = E_o \exp(i\omega t)$. Setting $\Delta l = \Delta l_0 \exp(i\omega t)$, the approximate solution for Δl_0 is given by

$$\Delta l_0 = \frac{P^s E_0}{2f_D} \tag{11}$$

Substituting the solution (11) into Eqs. (8) and (9), the induced strain $\Delta\varepsilon_{11}$ and polarization ΔP_3 by the domain wall motion may be written in the following form:

$$\Delta\varepsilon_{11} = \Delta d_{31}E_3 \tag{12}$$

$$\Delta P_3 = \Delta \in_{33} E_3 \tag{13}$$

where:

$$\Delta d_{31} = \frac{\gamma^s P^s}{2lf_D}, \quad \Delta \in_{33} = \frac{P^{s2}}{2lf_D} \tag{14}$$

Experimental studies on PZTs have shown that as much as 45–70% of dielectric and piezoelectric moduli values may originate from the extrinsic contributions [4,5]. Li et al. [10] approximately estimated $\Delta\varepsilon_{33}$ as two thirds of the measured value. Here, the extrinsic dielectric constant $\Delta\in_{33}$ is described by:

$$\Delta \in_{33} = \in_{33} \frac{2E_0}{3E_c} \tag{15}$$

where E_c is a coercive electric field.

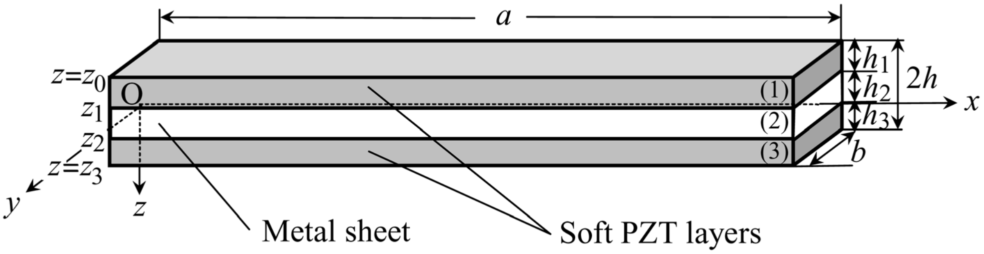

Fig. 2. Three-layered soft-type piezoelectric actuator.

2.2. Laminated beam theory

A laminated beam with integrated soft PZT layers is shown in Fig. 2. Let the coordinate axes $x = x_1$ and $y = x_2$ be chosen such that they coincide with the middle plane of the laminated actuator and the $z = x_3$ axis is perpendicular to this plane. The host material chosen is a metal. The same thick layers of PZT poled in z-direction are added to the upper and lower surfaces to make a three-layered piezoelectric actuator. The total thickness is $2h$ and the kth layer has thickness $h_k = z_k - z_{k-1}$ ($k = 1,2,3$) where $z_0 = -h$ and $z_3 = h$. For the present laminated beam theory, it is assumed that the electric field resulting from variations in stress (the so-called direct piezoelectric effect) is insignificant compared with the applied electric field [6].

The lamina constitutive equation for the kth layer of length a and width $b \ll a$ with respect to the reference axes of the laminate (x,z) can be expressed as:

$$(\sigma_{xx})_k = \frac{1}{(s_{11})_k}(\varepsilon_{xx})_k - \frac{(d_{31}^s)_k}{(s_{11})_k}(E_z)_k \tag{16}$$

where

$$(s_{11})_k = s_{11}, \quad (d_{31}^s)_k = d_{31} + \Delta d_{31} \quad (k = 1, 3) \tag{17}$$

$$(s_{11})_2 = s_{11}^E, \quad (d_{31}^s)_2 = 0 \tag{18}$$

and s_{11}^E is an elastic compliance of the metal. The bending modulus D and bending moment M_{xx}^E per unit length are given by:

$$D = \sum_{k=1}^{3} \int_{z_{k-1}}^{z_k} \frac{1}{(s_{11})_k} z^2 dz \tag{19}$$

$$M_{xx}^E = \sum_{k=1}^{3} \int_{z_{k-1}}^{z_k} \frac{(d_{31}^s)_k}{(s_{11})_k}(E_z)_k z dz \tag{20}$$

2.3. Displacement of cantilever laminated beam actuator

Consider the electroelastic response of a cantilever beam that is fixed at one end ($x = 0$) and subjected to an external ac electric field $E_z = E_0 \exp(i\omega t)$. The differential equation of motion for the displacement w can be expressed as:

$$Dw_{,xxxx} + 2h\rho^L w_{,tt} = 0 \tag{21}$$

where:

$$\rho^L = \sum_{k=1}^{3} \frac{\rho_k h_k}{2h} \tag{22}$$

The boundary conditions are:

$$\left.\begin{array}{ll} w = 0, \quad w_{,x} = 0 & (x = 0) \\ w_{,xx} = -\frac{M_{xx}^E}{D}, w_{,xxx} = 0 & (x = a) \end{array}\right\} \tag{23}$$

The displacement at $x = a$ is:

$$w = -\frac{\sin(ka)\sinh(ka)}{\{\cos(ka)\ \cosh(ka) + 1\}Dk^2} M_{xx}^E \exp(i\omega t) \tag{24}$$

where:

$$k^2 = \left(\frac{2h\rho^L}{D}\right)^{1/2} \omega \tag{25}$$

3. Finite element analysis

We performed three-dimensional finite element calculations to determine the displacement for the cantilever laminated actuators. We use the commercial finite element code ANSYS. Eight-node three-dimensional space solid was used in the analysis.

4. Experiment procedure

The present piezoelectric-shim-piezoelectric actuator was made of soft PZT and metal (Fuji Ceramics Ltd. Co. Japan). The piezoelectric PZT (C-91) had Ni paste electrodes on both sides. The metal layer was alloy sheet (Fe-42% Ni). The material properties of C-91 are listed in Table 1. The elastic compliance s_{11}^E and Poisson's ratio ν^E of metal sheet are taken to be $s_{11}^E = 4.76 \times 10^{-12}\ \mathrm{m^2/N}$

Table 1
Material properties of C-91

s_{11}	17.1×10^{-12} (m^2/V)	s_{12}	-6.3×10^{-12} (m^2/V)
s_{13}	-7.3×10^{-12} (m^2/V)	s_{33}	18.6×10^{-12} (m^2/V)
s_{44}	41.4×10^{-12} (m^2/V)	d_{31}	-340×10^{-12} (m/V)
d_{33}	645×10^{-12} (m/V)	d_{15}	836×10^{-12} (m/V)
$\in_{11}$	3.95×10^{-8} (F/m)	$\in_{33}$	4.90×10^{-8} (F/m)

and $\nu^E = 0.3$. The specimen had a length, a, of 40 mm, a width, b, of 2 mm, and a thickness, $2h$, of 0.64 mm. The amplitudes of the dynamic displacements of the cantilever actuators were measured with a microscope. A voltage was applied to the surface of the first piezoelectric layer.

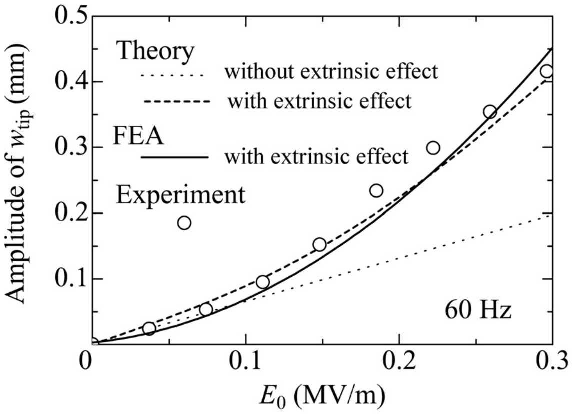

Fig. 3. Tip displacement versus electric field at 60 Hz.

5. Results and discussion

Figure 3 shows the amplitude of tip displacement w_{tip} as a function of the amplitude of electric field $(E_z)_1 = E_0 \exp(i\omega t)$ for $(E_z)_3 = 0$ V/m at 60 Hz. A nonlinear relationship between tip displacement and electric field is observed. Agreement between analysis and experiment is fair.

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