Model-adaptive structural FEM computations for fluid–structure interaction

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Abstract

A numerical approach to model thin-walled structures with anisotropic high-order finite hexahedral elements is presented. With an appropriate variation of the polynomial degrees an efficient discretization of a structural dynamic problem is found and then applied to a coupled fluid–structure interaction computation. Since a quasi-optimal choice of the polynomial degrees for every element as well as for every local direction of each element is highly desirable, an automatic p-adaptive method is presented and its effectivity is demonstrated with a numerical example.

Keywords: Model-adaptivity; FEM; P-adaptivity; Structural dynamics; Thin-walled structures; Fluid–structure interaction

1. Introduction

For many fluid–structure interaction problems it is desirable to avoid the use of dimensionally reduced theories for the structural model. Typical examples are structures with transitions from massive to thin-walled structures, e.g. shells with stiffeners, bridges on supports, rotor blades, etc. Here, finite elements based on dimensionally reduced models would necessitate the application of transition elements. Furthermore, at those transitions it is difficult to decide where a reduced stress state may be assumed and where an arbitrary three-dimensional stress state can occur. In these situations, fully three-dimensional numerical structural models are sought which are efficient for highly demanding coupled computations.

In Section 2 our approach with high-order hexahedral elements is described, which is utilized in Section 3 to find an efficient and accurate structural dynamic finite element model for a thin-walled structure by varying the polynomial degrees in the different local directions appropriately. The resulting numerical model is then used for a coupled fluid structure interaction computation. An automatic element-wise p-adaptive method is suggested in Section 4 and its efficiency is demonstrated by considering a linear elastostatic example.

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2. Structural simulation using three-dimensional, high-order finite elements

The problem usually arising when using a strictly three-dimensional approach for thin-walled parts of structures is that standard low-order finite elements are very sensitive to large aspect ratios, and locking effects are very likely to occur. Therefore, one would have to use a large number of small elements having a width and length in the same range as the thickness. In contrast, high-order elements can cope with high aspect ratios [1,3,11], provided that the polynomial degree is high enough.

In this article the use of high-order hexahedral elements for thin-walled structures is described [3,4,5]. Curved structures (see Fig. 1) can be taken into account by applying appropriate mapping techniques [4,6].

A very important feature in the context of high-order elements is anisotropic Ansatz spaces. Using the approach presented in [3] one can define different polynomial degrees in the different local directions of the hexahedral element. For a shell-like structure like that presented in Fig. 1, we apply a high polynomial degree for the in-plane direction, whereas in thickness direction a lower polynomial degree can be used in order to reduce the computational effort. In our implementation, it is possible to define different polynomial degrees not only for the different local directions, but also for the different components of the displacement field.

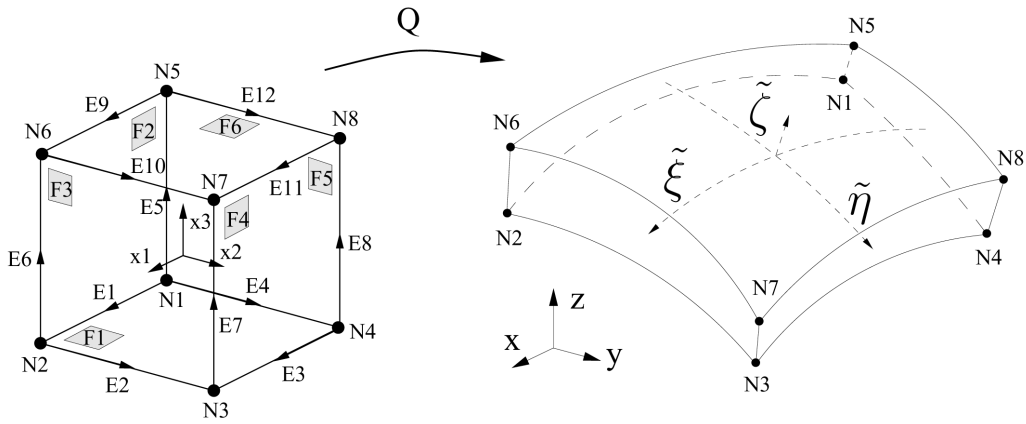

Fig. 1. Discretization of thin-walled structures with hexahedral elements.

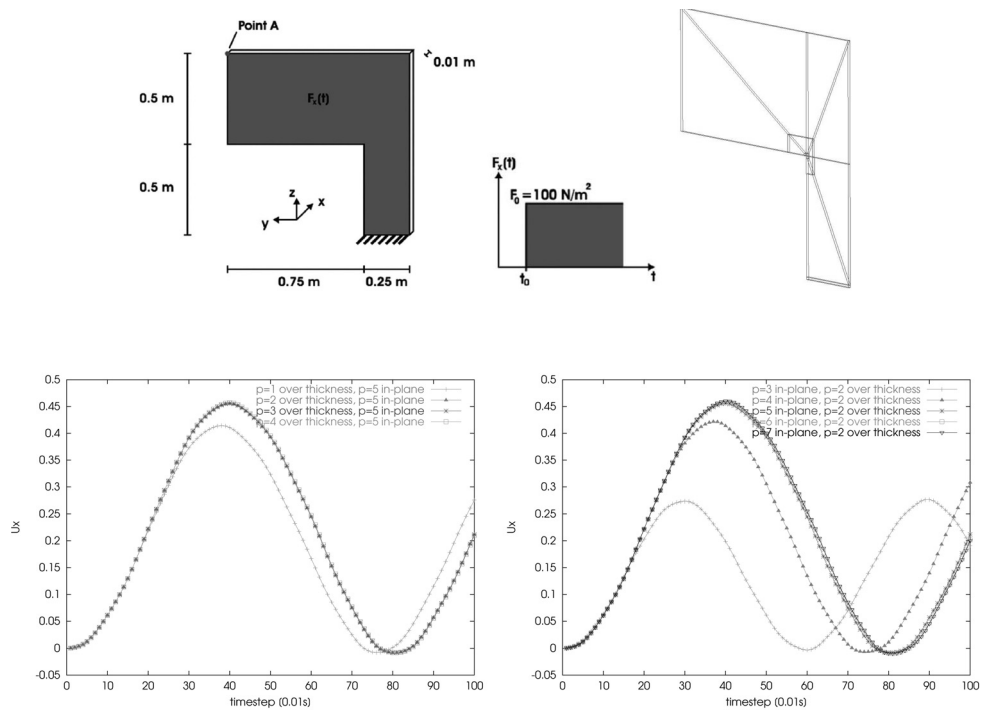

Fig. 2: System of plate with re-entrant corner (top left), mesh (top right), displacements of point A over time for varying q (bottom left) and p (bottom right).

Applying this approach, the 'model error' related to every plate or shell theory turns into the discretization error of the three-dimensional approximation. The main advantage is that this error can be controlled by varying the polynomial degree over the thickness in a sequence of computations. It should be mentioned that this error could not be controlled when using fixed kinematic assumptions. An example for such an approach to error control is shown in Section 3.

For the spatial discretization of the structural problem, high-order elements are used, whereas the time domain is discretized using the generalized-α-method, which is second-order accurate and has favorable numerical damping properties [7].

3. Error control for a thin-walled structure

As already mentioned in Section 2, the capability of error control by using high-order elements will be

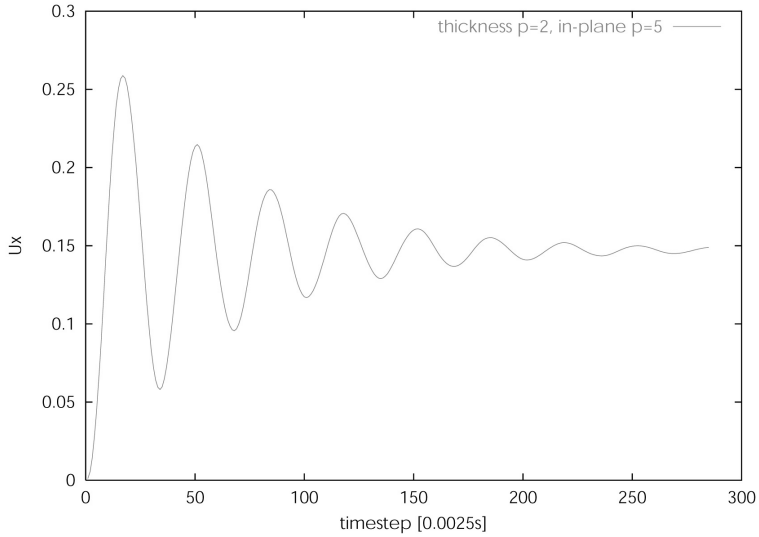

Fig. 3. Displacement u_x of point A over time (FSI).

demonstrated. Here, only the spatial error is considered, assuming that the discretization in time is sufficiently accurate.

For the system of a plate with a re-entrant corner subjected to a constant face load (see Fig. 2, top left), the error in the displacements for a structural dynamic, geometric nonlinear computation using a linear elastic material [8] is investigated.

The constant face load $F_x(t)$ acting on the shaded surface of the plate is applied suddenly at t_0. The plate is at rest and undeflected for $t \leq t_0$. Figure 2 (top right) depicts the mesh consisting of 16 hexahedrons, refined in order to resolve the singularity at the re-entrant corner.

In order to control the error, the following investigation was carried out. The displacement in x-direction of point A (see Fig. 2, top left) is considered over time. In a first step the polynomial degree in in-plane direction p is set to 5 and the degree in thickness direction q is varied in the range 1, ..., 4.

From Fig. 2 (bottom left) it is obvious that a polynomial degree of $q = 1$ does not yield a reliable result. However, if the polynomial degree in thickness direction is chosen to be $q \geq 2$, one obtains an accurate approximation.

In a second step, the polynomial degree in thickness direction is set to $q = 2$, and the polynomial degree in the in-plane direction p is varied in the range 3, ..., 7 (see Fig. 2, bottom right).

It can be summarized, that $p = 5$ and $q = 2$ yields a sufficiently accurate and efficient discretization for this problem.

This discretization is now used in a fluid–structure interaction framework [9] connecting our p-version structural code *AdhoC* and the block-structured finite volume fluid code *FASTEST3D* applying the code coupling software MpCCI [2] for interpolation between the non-matching grids. A very low inflow velocity of the fluid in x-direction was selected, resulting in a Reynolds number of $Re = 50$ (laminar flow).

Fig. 3 shows the oscillations of the corner node A (see system in Fig. 2) in x-direction. The amplitudes are large in the beginning, but are then decreasing due to the damping influence of the surrounding fluid.

4. An element-wise, automatic p-adaptive approach

In Section 3 there were only two free parameters in the adaptive computation, namely the polynomial degree in in-plane direction p and the polynomial degree over the thickness q of the *whole computational model*. In this section, we suggest an element-wise adaptive strategy for the computation of thin-walled structures with anisotropic hexahedral elements for a linear elastostatic problem and present some preliminary results, based on the weak form of equilibrium $\mathcal{B}(\mathbf{u}, \mathbf{v}) = \mathcal{F}(\mathbf{v})$. Future work will extend this procedure to elastodynamic problems.

In a first step we set the in-plane polynomial degree for all elements to, for example $p_k = 4$, and compute a hierarchic, (almost) locking-free sequence where q_k is adapted automatically (k denotes the element). Therefore, a hierarchic error indicator is applied, which compares the strain energy $\frac{1}{2}\mathcal{B}_k(\mathbf{u}_{FE}^{(p_k,q_k)}, \mathbf{u}_{FE}^{(p_k,q_k)})$ of each

element k for two different approximations. To obtain a reasonable error indicator, the approximation related to degree q_k is compared to the one based on $q_k + 2$ with

$$\lambda_k^2 = \frac{1}{2}\left|\mathcal{B}_k(\mathbf{u}_{FE}^{(p_k,q_k+2)}, \mathbf{u}_{FE}^{(p_k,q_k+2)}) - \mathcal{B}_k(\mathbf{u}_{FE}^{(p_k,q_k)}, \mathbf{u}_{FE}^{(p_k,q_k)})\right| \quad (1)$$

assuming that the saturation condition holds [10]. Having found the element-wise distribution of q_k we analogously adapt – in a second step – during an adaptive hierarchic refinement the polynomial degree p_k in in-plane. Therefore, q_k is kept constant and the strain energy is compared element-wise for different levels of p_k:

$$\lambda_k^2 = \frac{1}{2}\left|\mathcal{B}_k(\mathbf{u}_{FE}^{(p_k+2,q_k)}, \mathbf{u}_{FE}^{(p_k+2,q_k)}) - \mathcal{B}_k(\mathbf{u}_{FE}^{(p_k,q_k)}, \mathbf{u}_{FE}^{(p_k,q_k)})\right| \quad (2)$$

Those elements whose error is above a certain level are increased in polynomial degree. The adaptive run is stopped as soon as a prescribed tolerance is achieved or a maximum number of runs is exceeded. For a global error control we use the Richardson extrapolation [1].

Based on the adaptive approach described in this section, the computation of a partially loaded, clamped plate is carried out. The plate is loaded by a uniform pressure acting on the grey shaded area, see Fig. 4 (top left). In order to resolve boundary layers, the mesh is refined towards the boundary. The resulting element distribution of q_k from the first step after 5 iterations, and the corresponding distribution of p_k from the second step after 8 iterations are depicted in Fig. 4 (top center and right).

It is evident that the adaptive algorithm detects those elements which need to be refined in polynomial degree. Elements which are close to the clamped boundary, where the plate exhibits a three-dimensional stress state, are assigned to a higher polynomial degree than those elements which perform almost a pure rigid body rotation.

To judge the efficiency of the adaptive approach we consider the convergence of the error in energy norm of adaptive and uniform refinement, plotted against the number of degrees of freedom on a double logarithmic scale, see Fig. 4 (bottom). Of course, best results are obtained when both q_k and p_k are adaptively chosen.

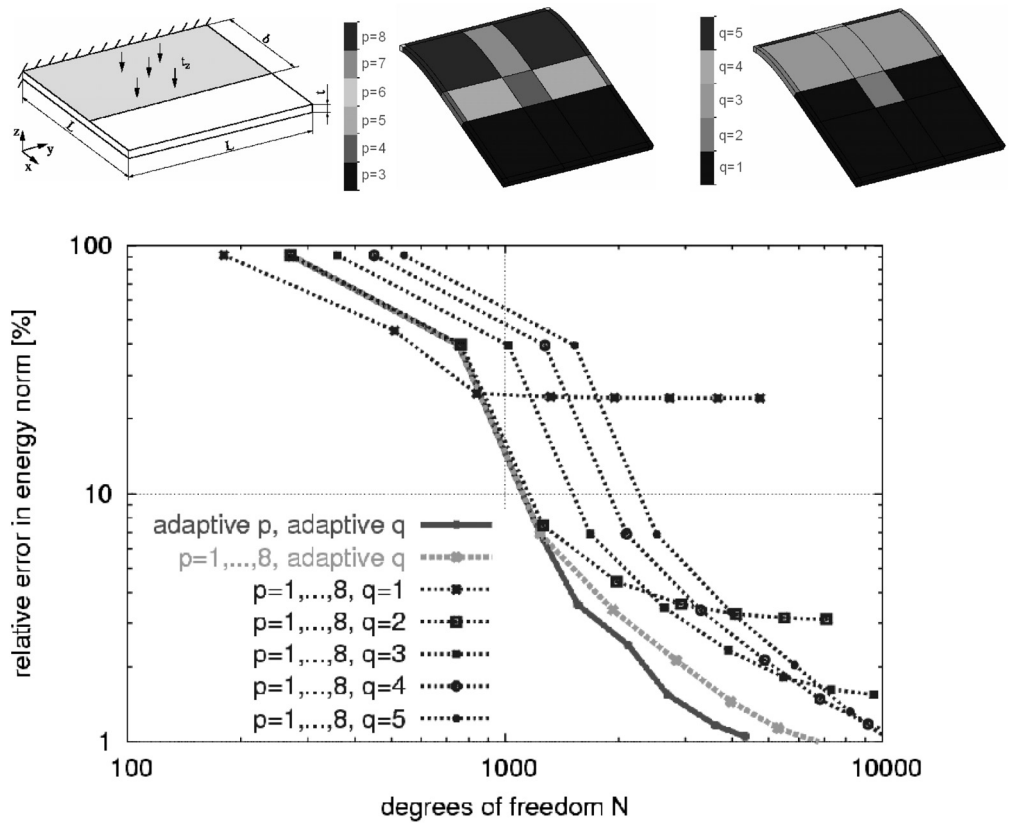

Fig. 4. Clamped plate (top left), adaptive selection of q_k (top middle) and p_k (top right), error in energy norm against n_{DOF} for uniform and adaptive computations (bottom).

However, already the pure q_k-adaption (model adaption) shows very good performance compared to uniform refinement.

5. Conclusions

Our fully three-dimensional approach for structural problems leads to very efficient finite element models by means of appropriate variation of the polynomial degree and p-adaptivity. These accurate but efficient models, especially of thin-walled structures, are then well suited e.g. for fluid–structure interaction computations as shown in the article. A disadvantage of the presented methods is their two-step character for thin-walled structures (first: adaption over thickness, second: adaption in-plane). Future work will consider this problem in more detail.

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