An embedded cohesive crack model for fracture of quasi-brittle materials

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Abstract

This paper presents a simple embedded crack model, based on the cohesive crack approach, for the fracture of quasi-brittle materials. A simple central force model is used to incorporate a cohesive softening curve (stress versus crack opening). The model requires only the elastic constants and a mode I softening curve. The need for a tracking algorithm is avoided by using a consistent procedure for the selection of the separated nodes. Numerical simulations of well-known experiments are presented.

Keywords: Cohesive crack; Concrete fracture; Finite element method; Mixed mode fracture; Numerical analysis; Embedded crack

1. Introduction

This work is based on the so-called *strong discontinuity approach* (SDA) to simulate the fracture of quasi-brittle materials [1]. A discrete constitutive model that relates the tractions and displacement jumps at the discontinuity line is used. A crack adaptation procedure avoids the need of the path enforcement (tracking).

2. The cohesive crack model

A simple generalization of the cohesive crack is used that assumes that the traction vector **t** transmitted across the crack faces is parallel to the crack displacement vector **w** (central forces model). For monotonic loading reads:

$$\mathbf{t} = \frac{f(|\tilde{w}|)}{\tilde{w}}\mathbf{w} \quad \text{with} \quad \tilde{w} = \max(|\mathbf{w}|) \tag{1}$$

where $f(|\tilde{w}|)$ is the classical softening function for pure opening mode and $\tilde{w}$ is the historical maximum of the magnitude of the crack displacement vector.

3. Finite element modelling

Figure 1a shows a finite element with an embedded crack. The crack splits the element in two sub-domains A^+ and A^-. **n** is the normal to one face and **w** is the displacement jump across the crack. The approximated displacement field can be written as:

$$\mathbf{u}(\mathbf{x}) = \sum_{\alpha \in A} N_\alpha(\mathbf{x})\mathbf{u}_\alpha + [H(\mathbf{x}) - N^+(\mathbf{x})]\mathbf{w} \tag{2}$$

where α is the element node index, $N_\alpha(\mathbf{X})$ is the shape function for node α, $\mathbf{u}_\alpha$ is the nodal displacement, and $H(\mathbf{x})$ is the Heaviside jump function across the crack plane. The strain tensor is obtained from the displacement field as a continuous part $\boldsymbol{\varepsilon}^c$ plus a Dirac's δ function on the crack line. The continuous part, which

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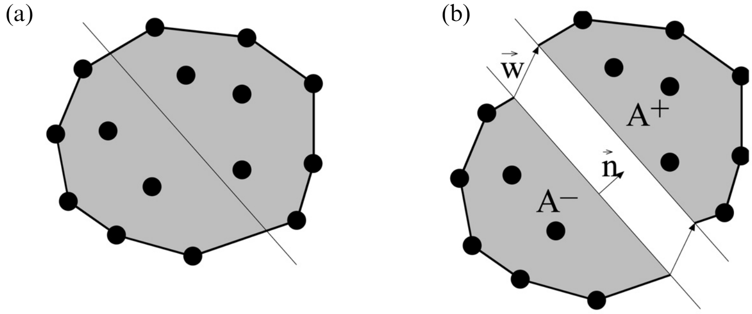

Fig. 1. Finite element with a crack with uniform opening: (a) generic element with nodes and crack line; (b) displacement jump across the crack line.

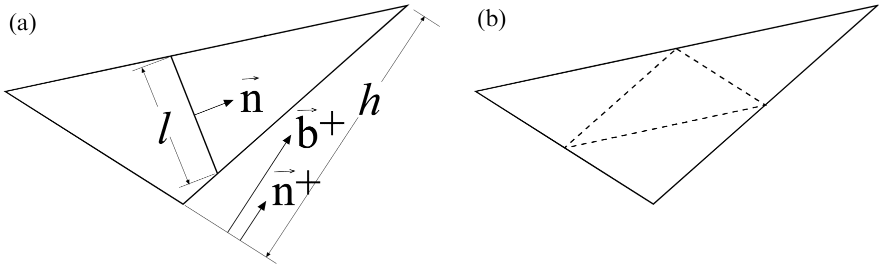

Fig. 2. Constant stress triangle: (a) geometrical definitions; (b) potential crack paths satisfying both local and global equilibrium (dashed lines).

determines the stress field on the element on both sides of the crack, is given by:

$$\boldsymbol{\varepsilon}^c(\mathbf{x}) = \varepsilon^a(\mathbf{x}) - [\mathbf{b}^+(\mathbf{x}) \otimes \mathbf{w}]^S \tag{3}$$

where ε^α and $\mathbf{b}^+$ are given by:

$$\boldsymbol{\varepsilon}^a(\mathbf{x}) = \sum_{\alpha \in A} [\mathbf{b}_\alpha(\mathbf{x}) \otimes \mathbf{u}_\alpha]^S \quad \text{and} \quad \mathbf{b}^+(\mathbf{x}) = \sum_{\alpha \in A^+} \mathbf{b}_\alpha(\mathbf{x}) \tag{4}$$

with $\mathbf{b}_\alpha$ $(\mathbf{x})$ = grad $N_\alpha(\mathbf{x})$. $\boldsymbol{\varepsilon}^\alpha$ is the *apparent* strain tensor computed from the nodal displacements. A constant strain triangle with a strong discontinuity line (crack), such as shown in Fig. 2, was developed. The uniform stresses in the crack are given by:

$$\mathbf{t} = \boldsymbol{\sigma} \cdot \mathbf{n} \quad \text{and} \quad \mathbf{t} = \frac{A}{hL} \boldsymbol{\sigma} \cdot \mathbf{n}^+ \tag{5}$$

where A is the area of the element, L is the length of the crack, h is the height of the triangle over the side opposite to the solitary node, and $\mathbf{n}^+$ is the unit normal to that side. The local equilibrium, Eq. (5), is used in conjunction with the strain approximation, Eq. (3).

4. Numerical implementation

The vector $\mathbf{w}$ is handled as two internal degrees of freedom, which are solved at the level of the crack within the finite element. From Eq. (3) and elastic bulk material behaviour, the stress tensor in the element is given by:

$$\boldsymbol{\sigma} = \mathbf{E} : \left[\boldsymbol{\varepsilon}^a - (\mathbf{b}^+ \otimes \mathbf{w})^S\right] \tag{6}$$

where $\mathbf{E}$ is the tensor of elastic moduli. To solve the crack displacement, the combination of Eqs (6), (5) and (2) leads to:

$$\left[\frac{f(\tilde{w})}{\tilde{w}} \mathbf{1} + \mathbf{n} \cdot \mathbf{E} \cdot \mathbf{b}^+\right] \cdot \mathbf{w} = [\mathbf{E} : \varepsilon^a] \cdot \mathbf{n} \tag{7}$$

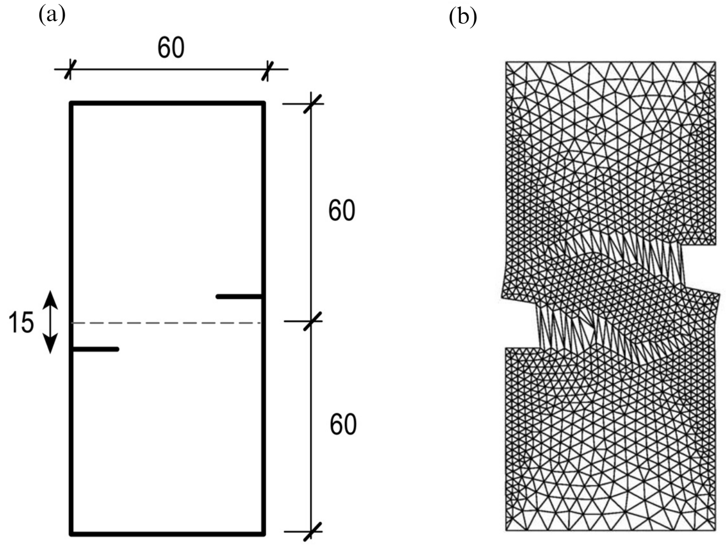

Fig. 3. (a) Geometry, forces and boundary conditions of the tests of Shi et al. [4]. (b) Deformed finite element mesh of the tests of Shi et al. [4]. Distances in millimetres.

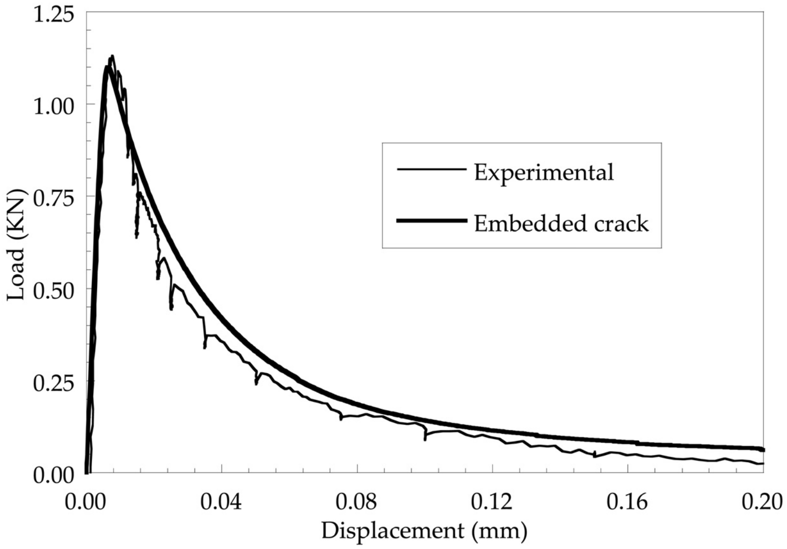

Fig. 4. Experimental envelope and numerical prediction of the tests of Shi et al. [4]: load–displacement curves.

This equation is solved for **w** using Newton Raphson's method given the nodal displacements (and so ε^a) once the crack is formed, and thus **n** and $\mathbf{b}^+$ are also given.

The crack is introduced perpendicular to the direction of the maximum principal stress, and **n** is computed as a unit eigenvector of $\boldsymbol{\sigma}$. The solitary node and $\mathbf{b}^+$ are determined by requiring the angle between **n** and $\mathbf{b}^+$ to be the smallest possible (see Fig. 2).

The foregoing procedure is done at the element level and is strictly local: no crack continuity is enforced or crack exclusion zone defined. This may lead to *locking* after a certain crack growth. To overcome this problem, a certain amount of crack adaptability within each element is allowed [2,3]. We allow the crack to adapt itself to later variations in principal stress direction while its opening is small. This crack adaptation is implemented easily by stating that while the equivalent crack opening at any particular element is less than a threshold value, the crack direction is recomputed at each step as if the crack were freshly created. After this threshold value, no further adaptation is allowed and the crack direction becomes fixed.

5. Numerical analysis of the mixed mode loading tests

The described model has been introduced in two commercial finite element codes by means of an user subroutine for material. The tests reported by Shi et al. [4] have been used as a benchmark for the numerical model. Fig. 3a shows the double-edge notched specimen subjected to direct tension, and Fig. 3b shows a deformed mesh, with finite elements with the embedded crack, used to simulate the fracture of the specimens. Fig. 4 shows the experimental results and the numerical prediction of the load P versus displacement curves. The peak load, the initial part of the curve and descending branch properly fit with experimental curve.

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