Localization analysis for overconsolidated Kaolin clay behavior

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Abstract

A numerical analysis of the shear band type strain localization is presented for a series of undrained triaxial compression tests on normally to highly overconsolidated kaolin clay. The concept of instability due to the singularity of acoustic tensor with the loss of strong ellipticity is applied in the analysis using a non-associative elastoplastic model that predicts the stress–strain relationship and pore pressure response reasonably well for the experimental data.

Keywords: Acoustic tensor; Elastoplasticity; Localization; Non-associative flow; Clay

1. Introduction

The theory of shear band type strain localization is a well accepted framework in geomechanics [1]. The critical shear band orientation and critical plastic hardening modulus is calculated based on the vanishing of the determinant of the acoustic tensor, which is derived from the tangential constitutive stiffness tensor [2]. This condition is also known as the loss of ellipticity, and the corresponding instability refers to the onset of shear band localization. An elastoplastic constitutive model with nonassociative flow rule yields asymmetric material stiffness tensor and thus acoustic tensors. Neilsen and Schreyer [3] and Szabó [4] presented mathematical formulations emphasizing that the loss of positive definiteness of the symmetric part of the acoustic tensor occurs before the loss of ellipticity. This condition is known as the loss of strong ellipticity. This paper presents a numerical evaluation of the loss of ellipticity and the loss of strong ellipticity using the experimental data from a series of undrained triaxial compression tests on normally to highly overconsolidated kaolin clay and a nonassociative constitutive model presented by Prashant and Penumadu [5]. The critical hardening modulus and the inclination of the critical planes of possible shear banding are evaluated in light of their variation with the overconsolidation level. In the following discussion, the tensor product is denoted by $\otimes$, and the following symbolic operations apply: $\mathbf{A}:\mathbf{B} = A_{ij}B_{ij}$, $(\mathbf{E}\cdot\hat{n})_{ijk} = E_{ijkl}n_l$, and $(\mathbf{E}:\mathbf{A})_{ij} = E_{ijkl}A_{kl}$, with the summation convention over repeated indices. The superposed dot denotes the material time derivative, or rate.

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2. Condition of continuous bifurcation in elastoplastic material

The classical elastoplasticity theory defines the following tangential constitutive relationships:

$$\text{Elasticity, } \dot{\sigma} = \mathbf{E} : \dot{\varepsilon} \tag{1}$$

$$\text{Elastic stiffness, } \mathbf{E}_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) \tag{2}$$

$$\text{Elastoplasticity, } \dot{\sigma} = \mathbf{D} : \dot{\varepsilon} \tag{3}$$

$$\text{Elastoplastic stiffness, } \mathbf{D} = \mathbf{E} - \frac{\mathbf{E} : \mathbf{P} \otimes \mathbf{Q} : \mathbf{E}}{H + \mathbf{Q} : \mathbf{E} : \mathbf{P}}$$

$$where, \mathbf{Q} = \frac{\partial f}{\partial \sigma}\Big/\left\|\frac{\partial f}{\partial \sigma}\right\| \quad and \quad \mathbf{P} = \frac{\partial g}{\partial \sigma}\Big/\left\|\frac{\partial g}{\partial \sigma}\right\|$$

Here, f is the yield function, g is the plastic potential, and $\|\cdot\|$ is the Euclidian norm of the tensor. The theory of localization defines the condition of continuous bifurcation for elastoplastic deformations across the shear band based on the vanishing of the determinant of acoustic tensor, which is derived from the constitutive stiffness tensor. For a unit vector $\hat{\mathbf{n}}$ normal to the shear band, the elastic acoustic tensor $\mathbf{B}^e$, and elastoplastic acoustic tensor $\mathbf{B}$ is defined using Eq. (6):

$$\mathbf{B}^e = \hat{\mathbf{n}} \cdot \mathbf{E} \cdot \hat{n} \text{ and } \mathbf{B} = \hat{\mathbf{n}} \cdot \mathbf{D} \cdot \hat{n} \tag{6}$$

The hardening modulus corresponding to the loss of ellipticity H_{le} is determined using Eq. (7):

$$H_{le} = (\mathbf{Q}:\mathbf{E}\cdot\hat{\mathbf{n}})\cdot\mathbf{E}^{-1}\cdot(\hat{\mathbf{n}}\cdot\mathbf{E}:\mathbf{P}) - (\mathbf{Q}:\mathbf{E}:\mathbf{P}) \quad (7)$$

The critical values of hardening modulus must obey the following condition:

$$H_{le} + (\mathbf{Q}:\mathbf{E}:\mathbf{P}) \geq 0 \quad (8)$$

For a nonassociative elastoplastic model ($f \neq g$), the stiffness tensor $\mathbf{D}$ and the acoustic tensors $\mathbf{B}$ are not symmetric, and the condition of bifurcation is defined by the vanishing of the symmetric part of the acoustic tensor (loss of strong ellipticity), det $\mathbf{B}_{sym} = 0$. In such condition, the hardening modulus H_{lse} is determined by using Eq. (9):

$$H_{lse} = \frac{1}{2}\left[\begin{array}{l}(\mathbf{Q}:\mathbf{E}\cdot\hat{\mathbf{n}})\cdot\mathbf{E}_{sym}^{-1}\cdot(\hat{\mathbf{n}}\cdot\mathbf{E}:\mathbf{P})+\\ \left\{(\mathbf{P}:\mathbf{E}\cdot\hat{\mathbf{n}})\cdot\mathbf{E}_{sym}^{-1}\cdot(\hat{\mathbf{n}}\cdot\mathbf{E}:\mathbf{P})\right\}^{1/2}\cdot\\ \left\{\left(\mathbf{Q}:\mathbf{E}\cdot\hat{\mathbf{n}}\right)\cdot\mathbf{E}_{sym}^{-1}\cdot(\hat{\mathbf{n}}\cdot\mathbf{E}:\mathbf{Q})\right\}^{1/2}\end{array}\right] - (\mathbf{Q}:\mathbf{E}:\mathbf{P}) \quad (9)$$

The occurrence of shear banding and its orientation is obtained by searching the largest critical hardening modulus and the corresponding unit vector $\hat{\mathbf{n}}$, which satisfy Eqs. (7) and/or (9).

3. A nonassociative elastoplastic model

The analysis presented in this paper is based on a nonassociative elastoplastic constitutive model developed by the authors. A detailed description of the proposed model can be found in Prashant and Penumadu [5]. Eqs. (11)–(14) show the key components of the model:

$$\textit{Yield Surface}: f = (q/p')^2 - L^2 \ln(p'_o/p') \quad (11)$$

$$\textit{Plastic Potential}\ \frac{\partial g}{\partial p'} = 2\left(\frac{p'}{p'_o}\right)^{\gamma} - 1, \quad \frac{\partial g}{\partial q} = n_g\left(\frac{\xi}{1-\xi}\right) \quad (12)$$

$$\textit{Mapping Variable}: \xi = q/q_y \quad (13)$$

$$\textit{Reference State Shear Stress}: q_y = C_y p'(p'_o/p')^{\Lambda_o} \quad (14)$$

Here, p' ($= I'_1/3$) is mean effective stress, p'_o is mean effective pre-consolidation stress, q ($= \sqrt{3J'_2}$) is deviatoric stress, L is a state variable, and γ, n_g, C_y, and Λ_o are material constants.

4. Analysis and results

The experimental data used in the following analysis of strain localization was obtained from a series of undrained triaxial compression tests on the specimens with a range of overconsolidation ratio (OCR) values.

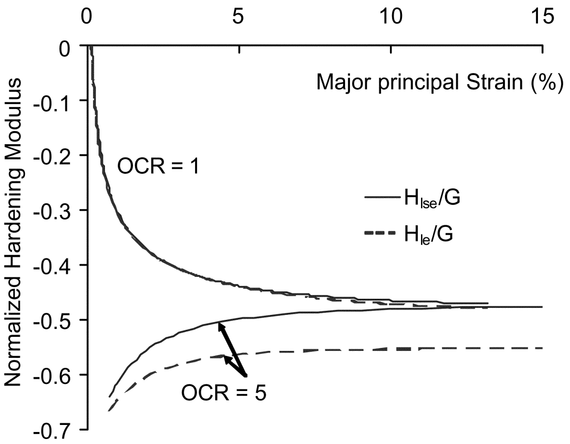

Fig. 1. Comparison between the loss of ellipticity and the loss of strong ellipticity.

Prashant and Penumadu [5] used the same experimental data, and reported that the observed stress–strain relationships and pore pressure response were reasonably predicted by the constitutive model used in this study. At each stress state, the critical hardening modulus was obtained by numerically solving the maximization problem for the rotation of vector $\hat{\mathbf{n}}$ in the $\sigma_1 - \sigma_3$ plane.

4.1. Loss of ellipticity vs. loss of strong ellipticity

The loss of ellipticity is defined by the singularity of acoustic tensor $\mathbf{B}$, and the loss of strong ellipticity by the singularity of $\mathbf{B}_{sym}$. Figure 1 shows variation of the normalized critical hardening modulus corresponding to the loss of ellipticity (H_{le}/G) and the loss of strong ellipticity (H_{lse}/G) during the undrained triaxial compression loading on normally consolidated (OCR = 1), and moderately overconsolidated (OCR = 5) kaolin clay. G is the elastic shear modulus of the pressure-dependent material. This figure shows that the values of H_{lse} for OCR = 1 case were not much different from the corresponding H_{le} values throughout shearing. However, H_{lse} values were generally higher than the values of H_{le} for both OCRs, indicating that the loss of strong ellipticity would occur before the loss of ellipticity, which is a common phenomenon for the models with the nonassociative flow rule [3]. Therefore, the condition of the loss of strong ellipticity is considered for further analysis of strain localization.

4.2. Critical hardening modulus and the angle of possible shear banding

Figure 2 shows a comparison between the plastic hardening modulus of the material and the critical

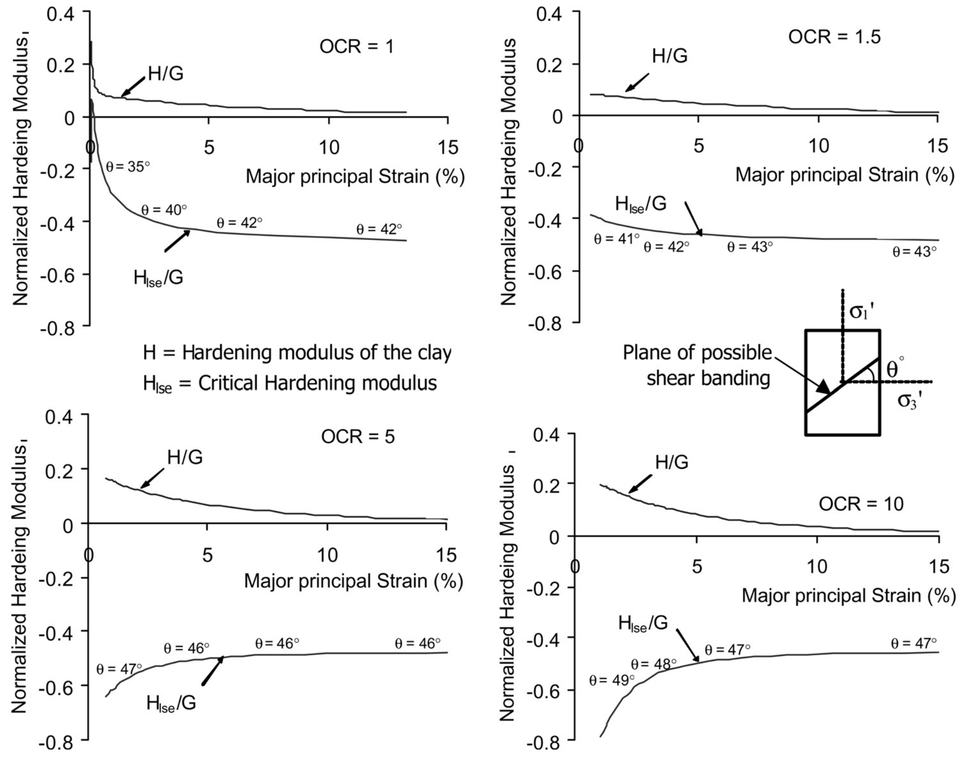

Fig. 2. Critical hardening modulus using the concept of the loss of strong ellipticity, and corresponding inclination of possible shear banding type strain localization.

hardening modulus H_{lse} corresponding to the loss of strong ellipticity for the undrained triaxial compression tests on the specimens with normally to highly overconsolidated kaolin clay (OCR = 1, 1.5, 5, and 10). This figure also shows the inclination of critical planes (θ) at various stress states. The values of H_{lse} were generally observed to be non-positive throughout shearing for all OCR values, and they were much lower than the material hardening modulus H. In the case of OCR = 1, the values of H_{lse}/G showed a sharp increase initially at very low strains and became positive; however, the H_{lse}/G values were consistently lower than the corresponding values of H/G. According to the theory of the vanishing of the acoustic tensor, the onset of shear band type strain localization in a soil element occurs when $H = H_{lse}$. This condition was never achieved during the hardening regime for the model to predict the onset of localization, which is consistent with the findings of Rudnicki and Rice [2] using a generalized and simple constitutive law for soils and rocks. Figure 2 shows that the variation of H_{lse}/G was significantly different for normally to lightly overconsolidated clay (OCR = 1 and 2) and moderately to highly overconsolidated clay (OCR = 5 and 10) due to change in the volumetric response of the soil from contractive to dilative with increasing OCR. However, the values of H_{lse}/G at large strains reached approximately a constant value of −0.48 for all OCR values. As shown in Fig. 2, direction of the plane representing the critical hardening modulus was calculated as θ-angle. For each OCR value, the values of θ-angle showed some variation at low strains but eventually they reached a constant value close to 5% of major principal strain. The final values of θ-angle at large strains showed a trend of increasing θ with OCR value, θ = 42° for OCR = 1 to θ = 47° for OCR = 10. For lightly to heavily overconsolidated clay, the critical θ-angle varies from one side to the other of the θ-angle corresponding to isotropic elastic material, which is θ = 45°. Although the critical H_{lse} values suggested that no shear band type localization would occur even at peak shear stress location, the θ-angle information may still be important because this angle provides the plane of maximum possible shear strains. It should also be noted that some other modes of instabilities might occur before shear banding, such as the growing non-uniformities due to undrained instability under globally undrained but locally drained conditions, [6].

5. Conclusions

The concept of instability based on the vanishing of the acoustic tensor was evaluated using the experimental data obtained from a series of undrained triaxial compression test on normally to highly overconsolidated kaolin clay. It was shown by a numerical evaluation that the loss of strong ellipticity would occur before the loss of ellipticity for non-associative materials. For all OCR cases, it was observed that a soil element subjected to undrained triaxial compression loading would not experience instability of the acoustic tensor before the experimentally observed failure location. However, the inclination of the possible weak planes could be important information from this analysis, which was shown to be a function of the OCR value.

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