Dynamic instability of circular cylindrical shells

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Abstract

In the present paper the nonlinear dynamic stability of circular cylindrical shells subjected to dynamic axial loads is investigated. Both Donnell's nonlinear shallow shell and Sanders' theories have been applied in order to evaluate their accuracy. The effect of a contained fluid on the dynamic stability and the postcritical behaviour is analysed in detail. Chaotic dynamics of compressed shells are investigated by means of nonlinear time series techniques, extracting correlation dimension and Lyapunov exponents.

Keywords: Shells; Instability; Vibration

1. Introduction

The development of aerospace vehicles requires deep studies of lightweight, thin-walled structures. A wide branch of the technical literature in the past century was focalized on the analysis of thin-walled structures and tried to investigate their behaviour in many different operating conditions; i.e. under static and dynamic loads, either in presence or absence of fluid-structure interaction.

The pioneer work on the stability of thin circular cylindrical shells date back to the middle of the previous century [1], and a nonlinear theory was applied in order to explain big discrepancies between linear theories and experiments. One of the first studies on parametrically excited shells can be found in [2]. In [3] it was observed that the classical membrane approach is inaccurate when the vibration contribution of the axisymmetric modes is not negligible. Recently, parametric instability of an infinitely long circular cylindrical shell was analysed [4,5,6].

The presence of geometric imperfections is an important factor to be considered in the development of a dynamical model of actual structures in operating conditions; such topic was considered in [7]. A review of studies on nonlinear dynamic stability and nonlinear vibrations of circular cylindrical shells is provided in [8]. Reference [9] is a report of a NATO project related to shells dynamics and stability with fluid structure interaction with several contributions.

In this work the static and dynamic behaviour of thin circular cylindrical shells subjected to axial dynamic load is considered. The Donnell's nonlinear shallow shell and Sanders' theories have been used. The displacement fields are expanded in series of linear eigenfunctions including asymmetric and axisymmetric modes. Geometric imperfections are included in the model and are normalized with respect to the shell thickness. The response of the shell subjected to static and periodic axial loads is investigated; a numerical solution of the governing equations is obtained by using a continuation technique. The dynamic analysis is concerned with the shell vibrations due to harmonic axial load at the ends, superimposed to a static axial preload.

2. Mathematical model and numerical results

In this section a short description of the mathematical models used to analyse the problem is provided. For the sake of brevity, the mathematical details are dropped and suitable references are provided.

Two shell theories are considered: Sanders' and Donnell's nonlinear shallow shell.

The Sanders' theory consists of a set of hyperbolic nonlinear partial differential equations in terms of the radial and in-plane displacement fields, w, u, v of the shell, see Fig. 1. This theory is based on Love's first

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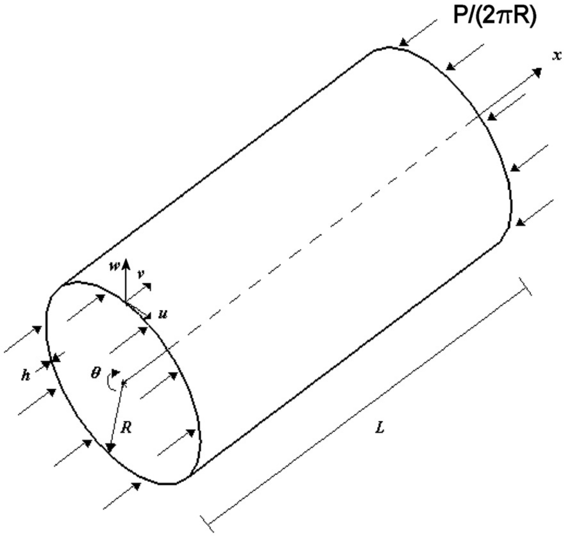

Fig. 1. Shell geometry, coordinate system and dimensions.

approximation: (i) $h \ll R$, h and R are the shell thickness and radius respectively; (ii) strains are small; (iii) transverse normal stress is small; and (iv) the normal to the undeformed middle surface remains straight and normal to the midsurface after deformation and undergoes no thickness stretching (Kirchhoff–Love kinematic hypothesis); rotary inertia and shear deformations are neglected [10].

The Donnell's nonlinear shallow shell theory consists of two nonlinear partial differential equations in terms of the radial displacement w and a stress function; indeed, the second partial differential equation is obtained from a static condensation of the in-plane equilibrium equations. In this theory the kinematics is simplified; moreover, the in-plane inertia is neglected. Therefore, the theory is accurate under the following additional hypotheses: the radial displacement is of the order of the shell thickness; the number of nodal diameters $n \geq 4$ [8,9].

In the case of a contained fluid the potential flow theory is considered; this allows to obtain the inertial fluid contribution in a closed form [9].

In the present work a benchmark problem, widely studied in the past, has been considered: $h = 2 \times 10^{-3}$ m, $R = 0.2$ m, $L = 0.4$ m, $E = 2.1 \times 10^{11}$ N/m^2. For this shell the fundamental buckling and the fundamental vibration modes have 5 nodal diameters and one longitudinal half wave ($m = 1$, $n = 5$).

When the shell is axially compressed it initially undergoes to an axial symmetric deformation, see Fig. 2; then, beyond $P/P_{cl} = 0.947$ $P_{\text{cl}} = 2\pi E h^2 \big/ \sqrt{3(1-\nu^2)}$), the shell loses stability; this load is the actual critical load (or bifurcation load).

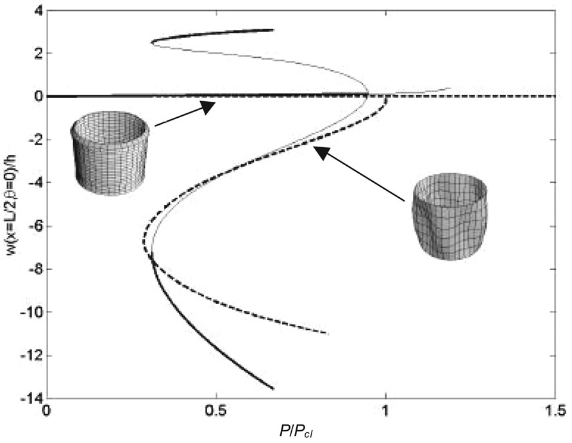

Fig. 2. Shell buckling: comparison between [6] '--' and the present theory (Sanders).

A first analysis is performed on a perfect shell by considering a purely harmonic axial load, i.e. no static preload is present: $\omega/\omega_{1,5(0)} = 1.9$ ($\omega_{1,5(0)} = 2 \times \pi$ 484.223 rad/s is the fundamental frequency of the shell without initial compression), the static axial load is $P = 0$, modal damping ratio $\zeta = 0.089$ on all modes; the dynamic axial load P_D is increased starting from zero excitation up to the onset of instability. Periodic solutions, their stability and bifurcations are studied by means of continuation techniques. When the amplitude of excitation is small, the shell vibrates axial-symmetrically with small amplitude, the response is periodic, as shown in Fig. 3. When $P_D/P_{cl} = 0.424$ (Sanders' theory) a period doubling bifurcation is found; increasing the dynamic load the solution becomes unstable. From the

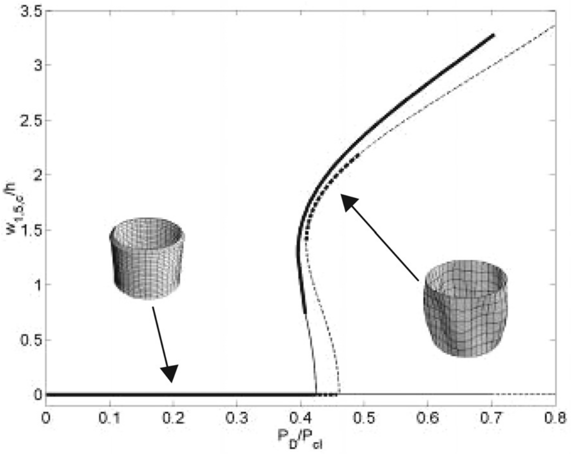

Fig. 3. Dynamic instability. '—' Sanders, '--' Donnell. Thick line: stable; thin line: unstable.

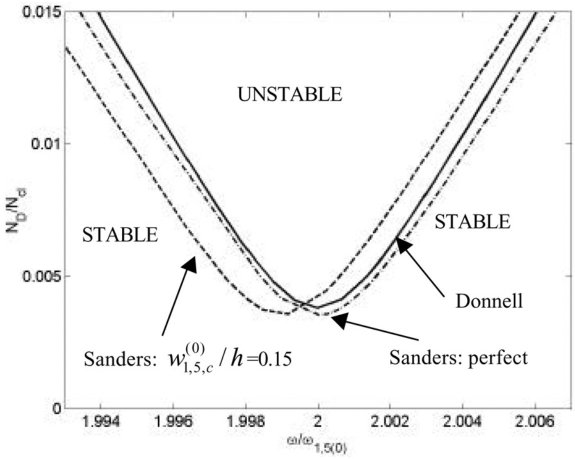

Fig. 4. Principal instability region: comparison of the two theories and effect of imperfections (on first mode).

bifurcation point a new solution takes place; it is slightly sub-critical and initially unstable; the response is no more axial symmetric, indeed, both asymmetric and axisymmetric modes are excited. In Fig. 3 results obtained by Donnell's nonlinear shallow shell theory are also shown; the bifurcation point is: $P_D/P_{cl} = 0.448$. The post-critical behaviour for both theories is in good agreement.

In Fig. 4 the principal instability region is presented for $\zeta = 0.0008$ and $P/P_{cl} = 0$: regions obtained with Donnell's nonlinear shallow shell and Sanders theories are very close; the effect of imperfection (modal imperfection $w^{(0)}_{1,5,c}/h = 0.15$) is a translation of the instability boundary, without changing the minimum value of P_D. A general conclusion is that small geometric imperfections, that give strong effects on the static buckling, are not effective on the parametric instability onset. In the following the symbol P_{Dcr} indicates the smallest dynamic amplitude for which one can obtain dynamic instability.

Table 2
Critical dynamic buckling: effect of fluid and comparison theories. $P/P_{cl} = 0$, damping ratio 0.089

$\omega/\omega_{1,5(0)}$	P_{Dcr}/P_{cl} (S–K theory)	P_{Dcr}/P_{cl} (Donnell)	Presence of water
2	0.704	0.722	yes
2	0.387	0.416	no

In Table 1 the effect of different imperfections is summarized: the general comment is that geometric imperfections are not quite effective on the parametric instability. The only case where the influence is evident regards a big imperfection (50% h) that gives rise to a growing of the critical dynamic load, which is mainly due to axial symmetric outward imperfections, that make the shell stiffer.

In the case of combined static and dynamic loads, chaotic dynamics can appear; for example for: $P/P_{cl} = 0.6$, $P_D/P_{cl} = 0.04$, $\omega/\omega_{1,5(0)} = 1.075$. By using embedding techniques and the Grassberger–Procaccia algorithm a correlation dimension equal to 3.5 is found; moreover, two positive Lyapunov exponents appear and the K–Y dimension is equal to 3.7.

In presence of fluid the scenario is changed; all natural frequencies are reduced and the dynamic critical load is increased, as shown in Table 2. The presence of fluid gives a safety effect.

3. Conclusions

The response of the shell structure under harmonic axial load shows a complex dynamics. Varying the frequency of the dynamic excitation, a direct resonance with softening behaviour and a parametric resonance appear. The critical dynamic load that causes the loss of stability of the shell with static preload is weakly affected by geometric imperfections; conversely, the imperfections

Table 1
Dynamic buckling: effect of imperfections (Sanders' theory if not indicated)

$\omega_{1,5}/\omega_{1,5(0)}$	$w^{(0)}_{1,5,c}/h$	$w^{(0)}_{1,15,c}/h$	$w^{(0)}_{3,5,c}/h$	$w^{(0)}_{1,0}/h$	$w^{(0)}_{3,0}/h$	P_{Dcr}/P_{cl}
1	0	0	0	0	0	0.0038 (Donnell)
1	0	0	0	0	0	0.0035
0.99091	0.1	0	0	0.1	0.1	0.0033
1.02912	0.1	0.1	0.1	0.1	0.1	0.0036
0.999748	0.1	0	0	0	0	0.0035
0.99117	0	0	0	0.1	0.1	0.0033
1.03715	0.1	0.1	0.1	0	0	0.0038
1.7001	0.5	0.5	0.5	0.5	0.5	0.0077
1.04611	−0.1	−0.1	−0.1	−0.1	−0.1	0.004
1.00975	0	0	0	−0.1	−0.1	0.0037

enlarge the frequency range in which the shell is dynamically unstable.

The presence of a contained fluid reduces the instability regions, i.e. gives a safety effect.

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