Non-linear numerical modeling of historical masonry on a meso-scale

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Abstract

This paper is concerned with the meso-scale modeling of mechanical response of regular natural stone masonry structures. The proposed approach relies on classical uncoupled numerical homogenization. The detailed analysis on a meso-scale is carried out to obtain the homogenized material parameters needed in the macroscopic constitutive law. Several examples are presented to document this modeling framework.

Keywords: Masonry; First-order homogenization; Fracture energy

1. Introduction

Nowadays, the engineering analysis of masonry structures draws upon a number of simplifying assumptions and phenomenological relations. This can be primarily attributed to the fact that masonry is a heterogenous material with a complex structure consisting of phases that exhibit quasi-brittle behavior. In the last decade, techniques of numerical first-order homogenization have been successfully used to obtain realistic models of regular masonry structures, in both the elastic, e.g. Anthoine [1], as well as inelastic ranges, Massart [2]. The most attractive feature of the homogenization-based approach is the fact that the non-linear behavior of the material, the initial anisotropy, together with its evolution due to progressive failure, directly follow from the analysis of a unit cell of a masonry structure rather than from a particular format of the macroscopic constitutive model.

2. Consistent numerical modeling

In this contribution, we limit our attention to regular natural stone masonry structures. A rebuilt arch of the Charles Bridge, Prague, shown in Fig. 1(a) is a typical example of such a structure suggesting a periodic character of the geometrical arrangement of individual stone blocks. The geometry of the mesoscopic periodic unit cell (PUC) for regular natural stone masonry structures is fully described by specifying the width and height of basic blocks, the thickness of head and bed joints, and the type of bond of a given structure. Such information can be easily obtained, e.g. by in-situ measurements or analysis of digital photographs of a structure. Having the representative value element (RVE) in a form of the aforementioned PUC, see Fig. 1(b), the analysis then follows a standard path set by the first-order homogenization as summarized in the next paragraph.

2.1. Homogenization and fracture energy (FE) analysis

Consider a heterogenous material sample, Y, subjected to a uniform strain, $\boldsymbol{E}$. Owing to the periodicity of the unit cell, the strain and displacement fields in the PUC admit the following decomposition:

$$\mathbf{u}(\mathbf{x}) = \boldsymbol{E} \cdot \mathbf{x} + \mathbf{u}^*(\mathbf{x}), \quad \varepsilon(x) = \boldsymbol{E} + \varepsilon^*(\mathbf{x}) \qquad (1)$$

The first term on the right-hand side of Eq. (1) corresponds to a displacement field in an effective homogeneous medium which has the same overall response as the heterogenous material. The fluctuating Y-periodic displacement $\mathbf{u}^*$ and corresponding strain ε^* enter Eq. (1) as a consequence of the presence of heterogeneities. The kinematic relations in Eq. (1), complemented with appropriate constitutive and balance laws, then constitute a well-defined unit cell

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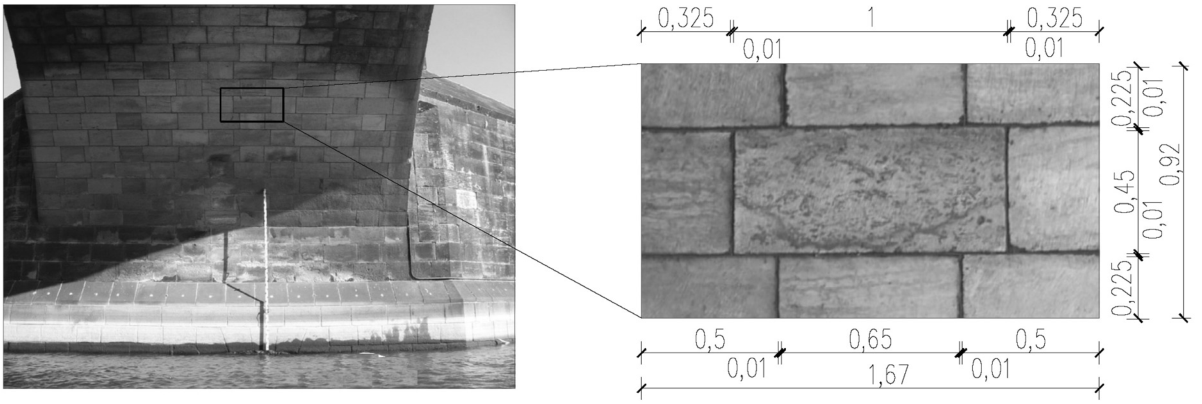

Fig. 1. (a) A rebuilt arch of the Charles Bridge; (b) RVE (representative volume element).

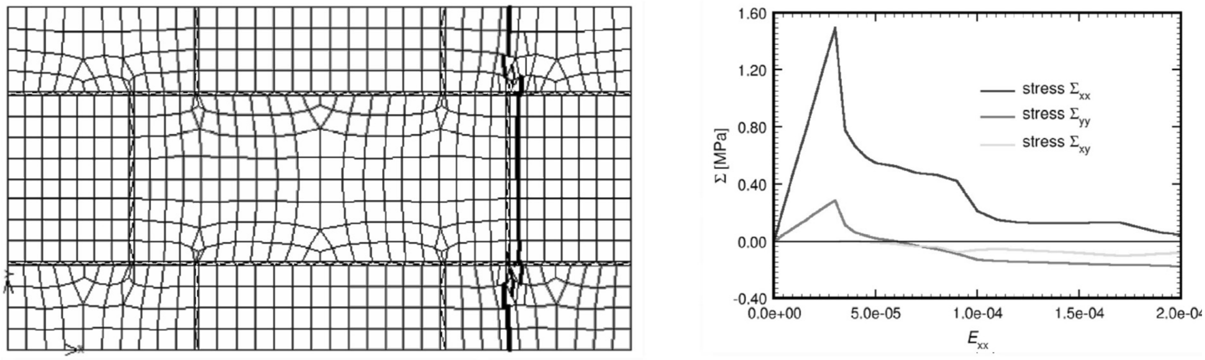

Fig. 2. (a) Cracked PUC after numerical simulation; (b) overall stress–strain curves.

problem that allows evaluation of the overall stress tensor $\boldsymbol{\Sigma}$ for a given overall strain $\boldsymbol{E}$, see, e.g. Michel et al. [3] and references therein. In the present work, the periodicity of the fluctuating displacement is ensured by tying the degrees of freedom related to displacements on the boundary of the unit cell (see Smit et al. [4] for more detailed discussion).

2.2. Numerical experiments

The main goal of the detailed analysis on a meso-scale is to estimate an effective response of a masonry while properly accounting for both the geometrical arrangement and material behavior of individual constituents (stone blocks and mortar joints). As already suggested, numerical experiments on a meso-scale provide the homogenized material response in form of macroscopic stress–strain curves. Arriving at these curves requires the introduction of a suitable constitutive model governing the response of individual constituents. The material phases exhibit a quasi-brittle behavior manifested by tensile softening attributed to the evolution of a localized zone of microcracks followed by formation of a discrete discontinuity with tractionless surfaces (see Fig. 2(a)). Such a state arises after full dissipation of strain energy in the tested material. In the present contribution such a complex material behavior is described by a plastic-fracturing model, NonLinearCementitious [5], implemented in the commercial software ATENA 2D. This model was used to describe the behavior of individual phases (for both blocks and mortar).

3. Conclusions

A simplified two-scale model is proposed to estimate the response of regular natural stone masonry. As an example, the procedure is applied to the analysis of the Charles Bridge in an attempt to assess the current state of damage. The meso-scale serves to determine the effective properties of masonry and non-linear loading paths ‘strain–stress curves’. Examples of such an analysis are presented in Fig. 2(b). The areas under these loading curves multiplied by the width of the localization band are proposed as the effective fracture energies for a forthcoming macro-scale analysis.

All of these numerical simulations can be realized immediately with the help of existing FE-codes (ATENA, ATENAWin).

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