Stochastic analysis of steady-state aeroelastic instabilities

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Abstract

A method for the numerical investigation of aeroelastic phenomena is proposed so that random properties of aerodynamic forces can be taken into account. The numerical analysis of aeroelastic instabilities is based on eigenvalue problems with random coefficients, thus enabling the analysis of the random characteristics of critical flow speeds, respectively. The aerodynamic coefficients are treated as normally distributed random variables. In order to determine the statistical properties of the critical speeds, a spectral stochastic finite element procedure is applied. Hence, the reliability of structures against steady-state aeroelastic instabilities can be estimated.

Keywords: Aeroelastic instabilities; Torsional divergence; Stochastic finite element method; Polynomial chaos; Stochastic eigenvalue problem; Random parameters

1. Introduction

In particular cases, the wind flow around engineering structures may cause vibrations with large amplitudes if the aerodynamic forces due to the fluid flow around the structure depend on the movement of the structure itself. Thus, the flow–structure interaction may induce self-excited vibrations leading to static aeroelastic instabilities, such as divergence phenomena, or to dynamic galloping or flutter instabilities. In order to enable numerical investigations, the vibration behavior is generally formulated in terms of eigenvalue problems, which allow the determination of critical flight speeds.

In engineering approaches, the aerodynamic forces are often expressed in terms of aerodynamic pressure coefficients. It is common practice to determine the coefficients in wind tunnel experiments. Then, averages of the measured values, which are fluctuating in space and time, are used for the numerical analysis of aeroelastic phenomena. When fluctuations are taken into account, eigenvalue problems with random parameters have to be regarded. A quite versatile method for the analysis of random eigenvalues and eigenvectors is the Monte-Carlo simulation. In order to reduce the numerical effort, several enhancements of the method have been proposed, e.g. for the numerical analysis of large systems by Pradlwarter et al. [1]. In [2], a polynomial chaos expansion is used for the description of the stochastic properties of the eigenvalues and eigenvectors. The coefficients are computed by Monte-Carlo simulation.

This paper outlines a spectral stochastic finite element method for the solution of the eigenvalue problem with stochastic parameters. The polynomial chaos expansion is used to represent the stochastic eigenvalues. Then, a Galerkin-like procedure is applied, leading to an enlarged system of multi-parametric eigenvalue equations. After some algebraic transformation, the eigenvalue equations are reduced to an ordinary eigenvalue problem of the size of the deterministic problem. Starting with this solution of the zeroth-order polynomial chaos expansion, the higher-order terms are determined afterwards.

2. Numerical investigation of aeroelastic instabilities

In order to present the solution method for the eigenvalue problem with random parameters, the steady-state aeroelastic stability of an airfoil is investigated in the following. If an airfoil with elastic properties is accidentally rotated in steady flight, an aeroelastic moment is generally induced, which causes a twisting of the airfoil. Since the moment due to the flow around the airfoil is proportional to the square of the flow speed, there exists a critical speed at which the

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elastic stiffness is annihilated and the airfoil is torsionally divergent.

The numerical investigations for fluid flow around the airfoil are based on the laws of potential flow theory. Thus, the action of the aerodynamic force can be represented by a lift force, which acts through the aerodynamic center, and a moment about the same point. When the equilibrium of forces and moments is formulated, the parts of aerodynamic force and moment, which depend on the angle of twist, can be inserted in an aerodynamic matrix **A**. Finally, the aerodynamic matrix couples the vertical and the twist deformation of the airfoil. When the stiffness properties, corresponding to vertical motion and the twist, are described by the matrix **K**, the divergence instability can be investigated by means of the eigenvalue problem

$$(\mathbf{K}-q(\theta)\mathbf{A}(\theta))\cdot\hat{\mathbf{y}}=\mathbf{0} \tag{1}$$

where stochastic variables are denoted by θ in case that random behavior of the fluid flow is taken into account. In equation (1), the impact pressure $q(\theta)$ remains as corresponding eigenvalue. Then, the critical speed u can be determined from $q = \frac{1}{2}\rho\,\mathrm{u}^2$, where ρ is the air density. In order to describe the random properties of the aeroelastic forces, the aeroelastic matrix is introduced by the sum $\mathbf{A}(\theta) = \mathbf{A}_0 + \xi(\theta)\,\mathbf{A}_1$. The sum consists of the mean value $\mathbf{A}_0$ and the random part with zero mean, which depends on the matrix $\mathbf{A}_1$ and the normally distributed random variable $\xi(\theta)$. As proposed by Spanos et al. [3], a polynomial chaos expansion is used to approximate the random impact pressure $q(\theta) = \sum_{\mathrm{j}=1}^{\mathrm{N}} \mathrm{q_j}\psi_{\mathrm{j}}(\theta)$, where $\psi_j(\theta)$ are Hermite polynomials in terms of the random variable $\xi(\theta)$ and q_j are deterministic coefficients, respectively. In order to determine the coefficients q_j, the eigenvalue Eq. (1) is weighted by each polynomial $\psi_m(\theta)$, resulting in a system of N eigenvalue equations. Finally, averaging $< \cdot >$ gives

$$[<\psi_m>\mathbf{K}+q_j(<\psi_m\psi^j>\mathbf{A}_0+<\xi\psi_m\psi^j>\mathbf{A}_1)]\cdot\hat{\mathbf{y}}=\mathbf{0} \tag{2}$$
$$m=1,\ \ldots,N,\quad j=1,\ \ldots,N$$

where the Einstein notation is used for better comprehension. The calculation of the statistical moments of polynomial expressions by numerical integration procedures may cause immense computational effort. However, analytical relations can be used, thus accelerating the averaging enormously. In order to determine the coefficients q_j and the eigenvector $\hat{\mathbf{y}}$, the system of eigenvalue Eq. (2) is at first resolved by means of triangular decomposition. Since the inverse of the aeroelastic matrices $\mathbf{A}_i$ have to be evaluated, the pseudo inverse is used, because $\mathbf{A}_i$ can be unsymmetric and singular. The triangular decomposition leads to an eigenvalue equation, in which the zeroth-order coefficient of the polynomial chaos expansion remains as corresponding eigenvalue. Thus, the eigenvalue problem can be solved for the zeroth-order coefficients and the eigenvectors. Then, the higher-order coefficients can be determined recursively, since the values for coefficients of lower order can be inserted in Eq. (2). In order to determine the higher order coefficients, it is more efficient to continue with a diagonalization of the eigenvalue equations using the eigenvector, which corresponds to the instability mode under consideration. Thus, the eigenvalue equations are reduced to a linear system of equations of the size of polynomial chaos expansion, which can be solved easily for all coefficients with respect to the instability mode.

As the triangular decomposition leads to an eigenvalue equation of the size of the deterministic problem, the approach is also well suited for the numerical investigation of structures, analyzed by finite element models with a large number of degrees of freedom. However, spatial fluctuations of the random field cannot be considered until now.

3. Divergence instability of an airfoil

The method is applied to the numerical investigation of an airfoil with a length of 11.96 m, a width of 1.15 m and a sweep angle of 5°. The bending stiffness is $EI = 2.78 \cdot 10^6$ Nm^2 and the shear stiffness follows to $GI_T = 2.41 \cdot 10^5$ Nm^2. The cantilever structure is investigated numerically by means of a finite element discretization of 40 elements, based on the Euler-Bernoulli beam and St. Venant torsion theory. The aerodynamic coefficient is approximately assumed to be $c_a = 2\pi$ [−], which can be analytically derived as parameter for a plane plate. Regarding these parameters, the deterministic analysis gives a critical speed of $u = 116.9$ m/s.

Figure 1 shows approximated probability density functions for the critical speed u, when random properties of the aerodynamic coefficients are taken into account. As presented for a standard deviation of $\sigma = 0.2$ [−], the spectral approach with a polynomial expansion up to the fourth order yields a good approximation for the probability density, compared to the results from a Monte-Carlo simulation. Figure 2 demonstrates that naturally the probability densities for u become more dispersed and skewed, if the standard deviation of the random coefficient increases. In order to approximate the tails of the probability density correctly in case of a standard deviation of $\sigma = 0.4$ [−], the polynomial chaos expansion is increased up to the seventh order. Furthermore, probabilities for chosen limit values of the critical speed are evaluated from the distributions, see Table 1. The probabilities give a quantitative measure for the appearance of the

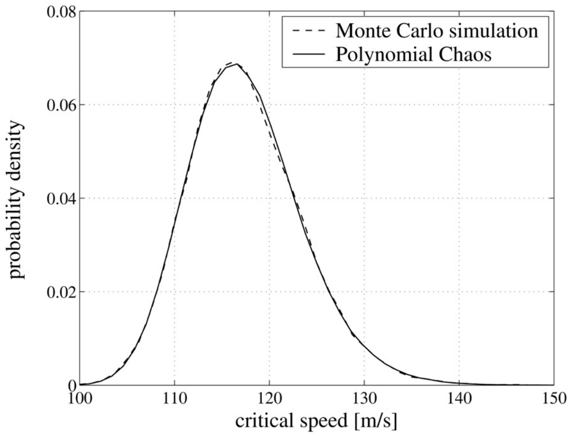

Fig. 1. Probability density functions for critical speed, $\sigma = 0.2\pi$.

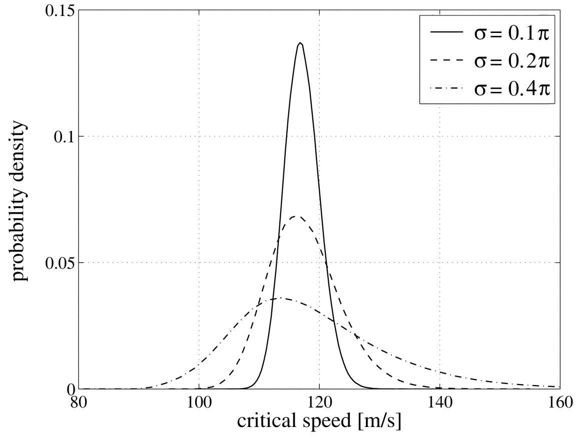

Fig. 2. Probability density functions for critical speed, different standard deviations σ.

divergence instability, thus allowing an estimation of the reliability.

Table 1
Probabilities for critical speed

	$\sigma = 0.1\pi$	$\sigma = 0.2\pi$	$\sigma = 0.4\pi$
$P(u \leq 100\,\mathrm{m/s})$	≈ 0	≈ 0	0.033
$P(u \leq 105\,\mathrm{m/s})$	≈ 0	0.008	0.111
$P(u \leq 110\,\mathrm{m/s})$	0.004	0.094	0.253
$P(u \leq 115\,\mathrm{m/s})$	0.239	0.361	0.431
$P(u \leq 120\,\mathrm{m/s})$	0.836	0.688	0.598

4. Conclusion

As demonstrated for the simple example of an airfoil, the probabilistic characteristics of critical flight speeds can be determined by means of a versatile method for eigenvalue problems with random parameters. Time-consuming computations, as compared to Monte-Carlo simulations, can be avoided.

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