Antiplane electro-mechanical field of a piezoelectric finite wedge under a pair of concentrated forces and free charges

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Abstract

This paper presents general antiplane electro-mechanical field solutions for a piezoelectric finite wedge subjected to a pair of concentrated forces and free charges. The boundary conditions on the circular segment are considered as traction free and insulated. Using the finite Mellin transforms method, the stress and electrical displacement at all fields of piezoelectric finite wedge are derived analytically; besides that, the singularity orders and intensity factors of stress and electrical displacement can be obtained too. After being reduced to the problem of an antiplane edge crack or an infinite wedge in a piezoelectric medium, the results compare well with those of previous studies.

Keywords: Antiplane problem; Piezoelectric finite wedge; Generalized intensity factor

1. Introduction

In the past, many researchers have used Mellin transforms to solve the elastic problem of a wedge shape. The wedge problems involving piezoelectric materials are rarely reported in literature. Xu and Rajapakse [1] discussed the inplane stress singularities of piezoelectric wedges. Chue and Chen [2] generalized Xu and Rajapakse's formulation to study the singularity orders of the piezoelectric wedges under generalized plane deformation. Chue et al. [3] carried out the singularity orders and generalized stress, strain, electrical field and electrical displacement intensity factors in a piezoelectric wedge under antiplane deformation by using Mellin transform.

Kargarnovin et al. [4] used finite Mellin transforms to obtain the displacement and stress components in an isotropic wedge with finite radius under antiplane deformation. They made a conclusion that the stress τ_{rz} and displacement w are divergent at the points of application of tractions. Furthermore, τ_{rz} is discontinuous on the arcs $r = h_1$ and $r = h_2$. These conclusions are incorrect. Obviously, the stress has to be continuous inside the wedge. Chue and Liu [5] have made a comment on it.

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2. Basic formulations and problem solutions

If the piezoelectric material is polarized along the z-axis, then the wedge problem will be decoupled to inplane and antiplane problems. The antiplane field couples the antiplane elastic deformation (τ_{xz}, τ_{yz}, w) and the inplane elastic parameters (D_x, D_y, E_x, E_y). In cylindrical coordinate system (r, θ), the constitutive equation of a piezoelectric medium polarized along the z-axis is given as:

$$\begin{bmatrix} \tau_{\theta z} \\ \tau_{rz} \\ D_r \\ D_\theta \end{bmatrix} = \begin{bmatrix} C_{44} & 0 & 0 & -e_{15} \\ 0 & C_{44} & -e_{15} & 0 \\ 0e & {}_{15} & \varepsilon_{11} & 0 \\ e_{15} & 0 & 0 & \varepsilon_{11} \end{bmatrix} \begin{bmatrix} \gamma_{\theta z} \\ \gamma_{rz} \\ E_r \\ E_\theta \end{bmatrix} \tag{1}$$

where τ_{ij} are the shear stress, γ_{ij} are the shear strain, D_i are the electric displacements and E_i are the electric field vectors. The material properties C_{44}, e_{15} and ε_{11} are the elastic stiffness constant, the piezoelectric constant and the dielectric constant, respectively. The shear strain-displacement and electric field–electric potential relations are:

$$\gamma_{\theta z} = \frac{1}{r}\frac{\partial w}{\partial \theta}, \quad \gamma_{rz} = \frac{\partial w}{\partial r}, \quad E_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}, \quad E_r = -\frac{\partial \phi}{\partial r} \tag{2}$$

where w and ϕ are displacement and electric potential, respectively. Substituting Eqs. (1) and (2) into static

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equilibrium equations and Maxwell's equation, the governing equations and solutions for w and ϕ are:

$$C_{44}\nabla^2 w + e_{15}\nabla^2\phi = 0, \quad e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2\phi = 0 \tag{3}$$

$$\nabla^2 w = 0, \quad \nabla^2\phi = 0 \tag{4}$$

Figure 1 shows a piezoelectric finite wedge with a wedge angle 2α and a finite radius a.

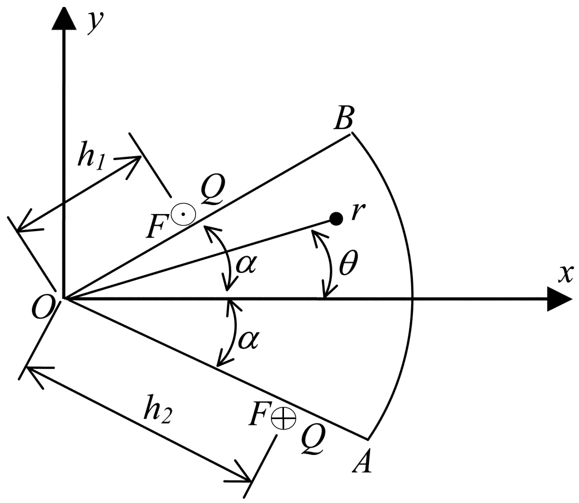


Fig. 1. A piezoelectric wedge with a wedge angle 2α and a finite radius a subjected to a pair of concentrated forces F and free charges Q.

The radial edges ($\theta = \pm\alpha$) are subjected to a pair of concentrated forces F and free charges Q:

$$\begin{aligned}\tau_{\theta z}(r,\alpha) &= F\delta(r-h_1)\\ \tau_{\theta z}(r,-\alpha) &= F\delta(r-h_2)\\ D_\theta(r,\alpha) &= Q\delta(r-h_1)\\ D_\theta(r,-\alpha) &= Q\delta(r-h_2)\end{aligned} \tag{5}$$

where $h_1 \le h_2$. The boundary conditions of the circular segment of the wedge ($r = a$) are assumed to be traction-free and electrically open. Applying the finite Mellin transform of the second kind to (4) gives:

$$\frac{\partial^2 w^*}{\partial\theta^2} + S^2 w^* = 0, \quad \frac{\partial^2\phi^*}{\partial\theta^2} + S^2\phi^* = 0 \tag{6}$$

provided that

$$\begin{aligned}&\left[\left(a^{2S}r^{-S+1} + r^{S+1}\right)\frac{\partial w}{\partial r} + \left(a^{2S}r^{-S} - r^S\right)Sw\right]_{r\to 0} = 0,\\ &\left[\left(a^{2S}r^{-S+1} + r^{S+1}\right)\frac{\partial \phi}{\partial r} + \left(a^{2S}r^{-S} - r^S\right)S\phi\right]_{r\to 0} = 0\end{aligned} \tag{7}$$

The solutions of (6) are:

$$\begin{aligned}w^* &= A(S)\cos S\theta + B(S)\sin S\theta,\\ \phi^* &= C(S)\cos S\theta + D(S)\sin S\theta\end{aligned} \tag{8}$$

where A, B, C and D are unknown functions of S and can be determined from Eqs. (5) and (8). By applying the inverse Mellin transform [4] on Eq. (8) and using the residue theorem and appropriate path of integration [4], w and ϕ can be obtained in three regions ($r \le h_1$, $h_1 \le r \le h_2$, and $a \ge r \ge h_2$). After using Eqs. (1) and (2) the stress and electric displacement are obtained except on the circular arcs $r = h_1$ and $r = h_2$. For example, the stress and electric displacement in $r < h_1$ are as follows:

$$\begin{aligned}&\tau_{rz}(r,\theta)\\ &= \frac{F}{2\alpha}\cdot\left\{\sum_{k=1}^{\infty}(-1)^k\left[\frac{1}{a}\left(\frac{r}{a}\right)^{\frac{k\pi}{\alpha}-1}\left(\left(\frac{h_1}{a}\right)^{\frac{k\pi}{\alpha}} - \left(\frac{h_2}{a}\right)^{\frac{k\pi}{\alpha}}\right)\right.\right.\\ &\quad\left. - \frac{1}{h_2}\left(\frac{r}{h_2}\right)^{\frac{k\pi}{\alpha}-1} + \frac{1}{h_1}\left(\frac{r}{h_1}\right)^{\frac{k\pi}{\alpha}-1}\right]\cos\left(\frac{k\pi\theta}{\alpha}\right)\\ &\quad + \sum_{k=0}^{\infty}(-1)^k\left[\frac{1}{a}\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}-1}\left(\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}} + \left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}\right)\right.\\ &\quad\left.\left. + \frac{1}{h_2}\left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{2\alpha}-1} + \frac{1}{h_1}\left(\frac{r}{h_1}\right)^{\frac{(2k+1)\pi}{2\alpha}-1}\right]\sin\left(\frac{(2k+1)\pi\theta}{2\alpha}\right)\right\}\end{aligned} \tag{9}$$

$$\begin{aligned}&\tau_{\theta z}(r,\theta)\\ &= \frac{F}{2\alpha}\cdot\left\{\sum_{k=1}^{\infty}(-1)^k\left[\frac{1}{a}\left(\frac{r}{a}\right)^{\frac{k\pi}{\alpha}-1}\left(\left(\frac{h_2}{a}\right)^{\frac{k\pi}{\alpha}} - \left(\frac{h_1}{a}\right)^{\frac{k\pi}{\alpha}}\right)\right.\right.\\ &\quad\left. + \frac{1}{h_2}\left(\frac{r}{h_2}\right)^{\frac{k\pi}{\alpha}-1} - \frac{1}{h_1}\left(\frac{r}{h_1}\right)^{\frac{k\pi}{\alpha}-1}\right]\sin\left(\frac{k\pi\theta}{\alpha}\right)\\ &\quad + \sum_{k=0}^{\infty}(-1)^k\left[\frac{1}{a}\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}-1}\left(\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}} + \left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}\right)\right.\\ &\quad\left.\left. + \frac{1}{h_2}\left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{2\alpha}-1} + \frac{1}{h_1}\left(\frac{r}{h_1}\right)^{\frac{(2k+1)\pi}{2\alpha}-1}\right]\cos\left(\frac{(2k+1)\pi\theta}{2\alpha}\right)\right\}\end{aligned} \tag{10}$$

$$\begin{aligned}&D_r(r,\theta)\\ &= \frac{Q}{2\alpha}\cdot\left\{\sum_{k=1}^{\infty}(-1)^k\left[\frac{1}{a}\left(\frac{r}{a}\right)^{\frac{k\pi}{\alpha}-1}\left(\left(\frac{h_1}{a}\right)^{\frac{k\pi}{\alpha}} - \left(\frac{h_2}{a}\right)^{\frac{k\pi}{\alpha}}\right)\right.\right.\\ &\quad\left. - \frac{1}{h_2}\left(\frac{r}{h_2}\right)^{\frac{k\pi}{\alpha}-1} + \frac{1}{h_1}\left(\frac{r}{h_1}\right)^{\frac{k\pi}{\alpha}-1}\right]\cos\left(\frac{k\pi\theta}{\alpha}\right)\\ &\quad + \sum_{k=0}^{\infty}(-1)^k\left[\frac{1}{a}\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}-1}\left(\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}} + \left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}\right)\right.\\ &\quad\left.\left. + \frac{1}{h_2}\left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{2\alpha}-1} + \frac{1}{h_1}\left(\frac{r}{h_1}\right)^{\frac{(2k+1)\pi}{2\alpha}-1}\right]\sin\left(\frac{(2k+1)\pi\theta}{2\alpha}\right)\right\}\end{aligned} \tag{11}$$

$$
\begin{aligned}
&D_\theta(r,\theta)\\
&=\frac{Q}{2\alpha}\cdot\left\{\sum_{k=1}^{\infty}(-1)^k\left[\frac{1}{a}\left(\frac{r}{a}\right)^{\frac{k\pi}{\alpha}-1}\left(\left(\frac{h_2}{a}\right)^{\frac{k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{k\pi}{\alpha}}\right)\right.\right.\\
&\left.+\frac{1}{h_2}\left(\frac{r}{h_2}\right)^{\frac{k\pi}{\alpha}-1}-\frac{1}{h_1}\left(\frac{r}{h_1}\right)^{\frac{k\pi}{\alpha}-1}\right]\sin\left(\frac{k\pi\theta}{\alpha}\right)\\
&+\sum_{k=0}^{\infty}(-1)^k\left[\frac{1}{a}\left(\frac{r}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}-1}\left(\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}\right)\right.\\
&\left.\left.+\frac{1}{h_2}\left(\frac{r}{h_2}\right)^{\frac{(2k+1)\pi}{2\alpha}-1}+\frac{1}{h_1}\left(\frac{r}{h_1}\right)^{\frac{(2k+1)\pi}{2\alpha}-1}\right]\cos\left(\frac{(2k+1)\pi\theta}{2\alpha}\right)\right\}
\end{aligned}
\tag{12}
$$

The singularity order for stresses and electrical displacements is $(\pi/2\alpha)-1$ when $-1 < (\pi/2\alpha)-1 < 0$. The singularity order is independent of the wedge radius a and coincides with the result of Chue et al. [3] for infinite wedge problem. No singularities are observed for $\alpha \leq \pi/2$. In addition, the order becomes conventional square root when the wedge structure becomes a crack in a piezoelectric medium, i.e. $\alpha = \pi$.

Substituting $r = h_1$ in Eqs. (9)–(12), the stress and electrical displacement at $r = h_1$ become:

$$
\begin{aligned}
&\tau_{rz}(h_1,\theta)\\
&=\frac{F}{2h_1\alpha}\cdot\left\{\sum_{k=1}^{\infty}(-1)^k\left[-\left(\frac{h_2}{a}\right)^{\frac{k\pi}{\alpha}}\left(\frac{h_1}{a}\right)^{\frac{k\pi}{\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{h_2}\right)^{\frac{k\pi}{\alpha}}\right]\right.\\
&\cos\left(\frac{k\pi\theta}{\alpha}\right)+\sum_{k=0}^{\infty}(-1)^k\left[\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right.\\
&\left.\left.+\left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{2\alpha}}\right]\sin\left(\frac{(2k+1)\pi\theta}{2\alpha}\right)-\frac{1}{2}\right\}
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&D_r(h_1,\theta)\\
&=\frac{Q}{2h_1\alpha}\cdot\left\{\sum_{k=1}^{\infty}(-1)^k\left[-\left(\frac{h_2}{a}\right)^{\frac{k\pi}{\alpha}}\left(\frac{h_1}{a}\right)^{\frac{k\pi}{\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}-\left(\frac{h_1}{h_2}\right)^{\frac{k\pi}{\alpha}}\right]\right.\\
&\cos\left(\frac{k\pi\theta}{\alpha}\right)+\sum_{k=0}^{\infty}(-1)^k\left[\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right.\\
&\left.\left.+\left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{2\alpha}}\right]\sin\left(\frac{(2k+1)\pi\theta}{2\alpha}\right)-\frac{1}{2}\right\}
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&\tau_{\theta z}(h_1,\theta)\\
&=\frac{F}{2h_1\alpha}\cdot\left\{\sum_{k=1}^{\infty}(-1)^k\left[\left(\frac{h_2}{a}\right)^{\frac{k\pi}{\alpha}}\left(\frac{h_1}{a}\right)^{\frac{k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}+\left(\frac{h_1}{h_2}\right)^{\frac{k\pi}{\alpha}}\right]\right.\\
&\sin\left(\frac{k\pi\theta}{\alpha}\right)+\sum_{k=0}^{\infty}(-1)^k\left[\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right.\\
&\left.\left.+\left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{2\alpha}}\right]\cos\left(\frac{(2k+1)\pi\theta}{2\alpha}\right)+\frac{\cos\left(\frac{\pi\theta}{4\alpha}\right)+\sin\left(\frac{\pi\theta}{4\alpha}\right)}{2\cos\left(\frac{\pi\theta}{4\alpha}\right)-2\sin\left(\frac{\pi\theta}{4\alpha}\right)}\right\},\\
&\theta\neq\pm\alpha
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&D_\theta(h_1,\theta)\\
&=\frac{Q}{2h_1\alpha}\cdot\left\{\sum_{k=1}^{\infty}(-1)^k\left[\left(\frac{h_2}{a}\right)^{\frac{k\pi}{\alpha}}\left(\frac{h_1}{a}\right)^{\frac{k\pi}{\alpha}}-\left(\frac{h_1}{a}\right)^{\frac{2k\pi}{\alpha}}+\left(\frac{h_1}{h_2}\right)^{\frac{k\pi}{\alpha}}\right]\right.\\
&\sin\left(\frac{k\pi\theta}{\alpha}\right)+\sum_{k=0}^{\infty}(-1)^k\left[\left(\frac{h_2}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{2\alpha}}+\left(\frac{h_1}{a}\right)^{\frac{(2k+1)\pi}{\alpha}}\right.\\
&\left.\left.+\left(\frac{h_1}{h_2}\right)^{\frac{(2k+1)\pi}{2\alpha}}\right]\cos\left(\frac{(2k+1)\pi\theta}{2\alpha}\right)+\frac{\cos\left(\frac{\pi\theta}{4\alpha}\right)+\sin\left(\frac{\pi\theta}{4\alpha}\right)}{2\cos\left(\frac{\pi\theta}{4\alpha}\right)-2\sin\left(\frac{\pi\theta}{4\alpha}\right)}\right\},\\
&\theta\neq\pm\alpha
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&\tau_{\theta z}(h_1,\alpha)=D_\theta(h_1,\alpha)\to\infty,\\
&\quad\tau_{\theta z}(h_1,-\alpha)=D_\theta(h_1,-\alpha)\to 0
\end{aligned}
\tag{17}
$$

Similarly, the stress and electrical displacement at $r = h_2$ can be obtained too. The distributions of stress and electrical displacement near the singular point of the wedge can be written as;

$$
\tau_{\theta z}(r,\theta)=K^{\tau}_{III}\cdot r^{\lambda-1}\cdot f_{\theta z}(\theta),\quad \tau_{rz}(r,\theta)=K^{\tau}_{III}\cdot r^{\lambda-1}\cdot f_{rz}(\theta)
\tag{18}
$$

$$
D_\theta(r,\theta)=K^{D}_{III}\cdot r^{\lambda-1}\cdot g_\theta(\theta),\quad D_r(r,\theta)=K^{D}_{III}\cdot r^{\lambda-1}\cdot g_r(\theta)
\tag{19}
$$

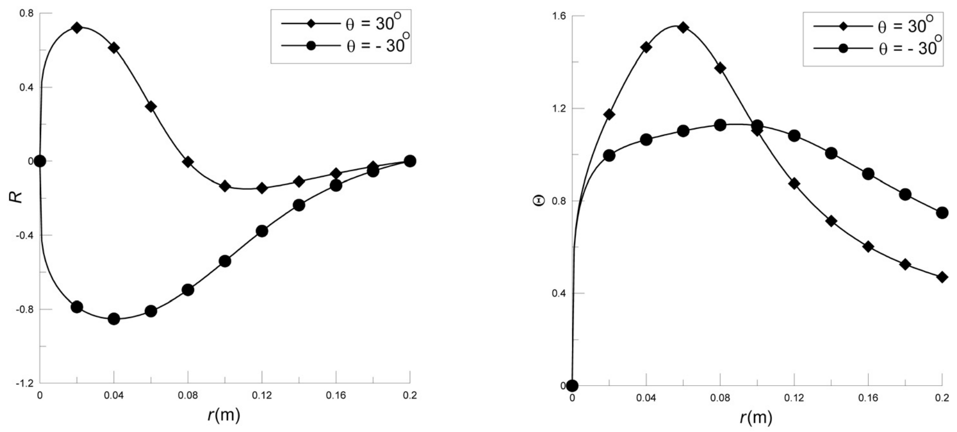
where $\mathrm{Re}[\lambda-1]$ is the singularity order, K^{τ}_{III} the generalized stress intensity factor, K^{D}_{III} the generalized electrical displacement intensity factor, and $f_{ij}(\theta)$ and $g_i(\theta)$ are the angular functions. Comparing Eqs. (18) and (19) with Eqs. (9)–(12), we obtain the intensity factors:

$$
K^{\tau}_{III}=\frac{\sqrt{2\pi}F}{2\alpha}\cdot\left[\left(\frac{h_2}{a^2}\right)^{\frac{\pi}{2\alpha}}+\left(\frac{h_1}{a^2}\right)^{\frac{\pi}{2\alpha}}+\left(\frac{1}{h_2}\right)^{\frac{\pi}{2\alpha}}+\left(\frac{1}{h_1}\right)^{\frac{\pi}{2\alpha}}\right]
\tag{20}
$$

$$
K^{D}_{III}=\frac{\sqrt{2\pi}Q}{2\alpha}\cdot\left[\left(\frac{h_2}{a^2}\right)^{\frac{\pi}{2\alpha}}+\left(\frac{h_1}{a^2}\right)^{\frac{\pi}{2\alpha}}+\left(\frac{1}{h_2}\right)^{\frac{\pi}{2\alpha}}+\left(\frac{1}{h_1}\right)^{\frac{\pi}{2\alpha}}\right]
\tag{21}
$$

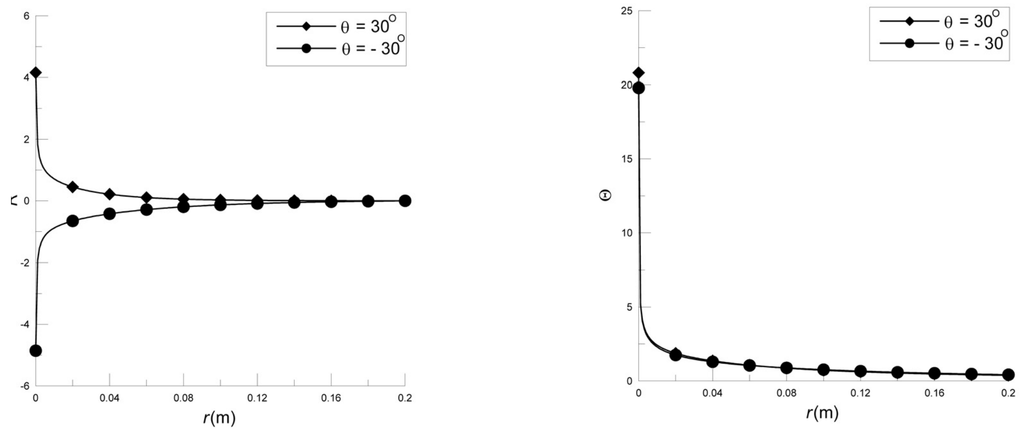
3. Results and discussions

We define $R(r, \theta) = \tau_{rz}(a/F) = D_r(a/Q)$ and $\Theta(r, \theta) = \tau_{\theta z}(a/F) = D_\theta(a/Q)$ as the normalized stresses and electrical displacements, respectively. When $a = 0.2$m, $h_1 = 0.08$m, $h_2 = 0.14$m, and $2\alpha = 150°$, Figs. 2(a) and 2(b) plot the distributions of R and Θ along $\theta = \pm 30°$, respectively. Since the singularities disappear for $2\alpha \leq 180°$, the stresses and electrical displacements remain in



(a) (b)

Fig. 2. Normalized stress and electrical displacement distributions when $a = 0.2$m, $h_1 = 0.08$m, $h_2 = 0.14$m, and $2\alpha = 150^\circ$ (a) $R = \tau_{rz}(a/F) = D_r(a/Q)$, (b) $\Theta = \tau_{\theta z}(a/F) = D_\theta(a/Q)$.



(a) (b)

Fig. 3. Normalized stress and electrical displacement distributions when $a = 0.2$m, $h_1 = 0.08$m, $h_2 = 0.14$m, and $2\alpha = 270^\circ$ (a) $R = \tau_{rz}(a/F) = D_r(a/Q)$, (b) $\Theta = \tau_{\theta z}(a/F) = D_\theta(a/Q)$.

finite values. It can be also seen that R and Θ are continuous across the circular arcs $r = h_1$ and $r = h_2$. In addition, R (i.e. τ_{rz} and D_r) vanishes along circular edge $r = a$ according to boundary conditions.

The results of R and Θ for case $2\alpha = 270^\circ$ are plotted in Fig. 3. It appears that the stresses and displacements approach infinity as $r \to 0$.

The normalized intensity factor K^* is defined as follows:

$$K* = K_{III}^{\tau} h_2^{\frac{\pi}{2\alpha}}/F = K_{III}^{D} h_2^{\frac{\pi}{2\alpha}}/Q = \sqrt{\frac{\pi}{2}}\frac{1}{\alpha}\left[\left(\frac{h_2}{a}\right)^{\frac{\pi}{\alpha}} + \left(\frac{h_2 h_1}{a^2}\right)^{\frac{\pi}{2\alpha}} + 1 + \left(\frac{h_2}{h_1}\right)^{\frac{\pi}{2\alpha}}\right] \qquad (22)$$

Figure 4 plots the variations of normalized intensity factor K^* with half wedge angle at different finite radius a when $h_1 = 0.08$m, $h_2 = 0.14$m. It shows that a finite wedge with smaller wedge angle 2α and smaller radius a results in larger generalized intensity factor. For an infinite piezoelectric wedge (i.e. $a \to \infty$) with $h = h_1 = h_2$ the generalized intensity factors for stress and electrical displacement become:

$$K_{III}^{\tau} = \frac{F}{\alpha}\sqrt{\frac{2\pi}{h^{\frac{\pi}{\alpha}}}} \qquad (23)$$

$$K_{III}^{D} = \frac{Q}{\alpha}\sqrt{\frac{2\pi}{h^{\frac{\pi}{\alpha}}}} \tag{24}$$

which compare well with those of Chue et al. [3]

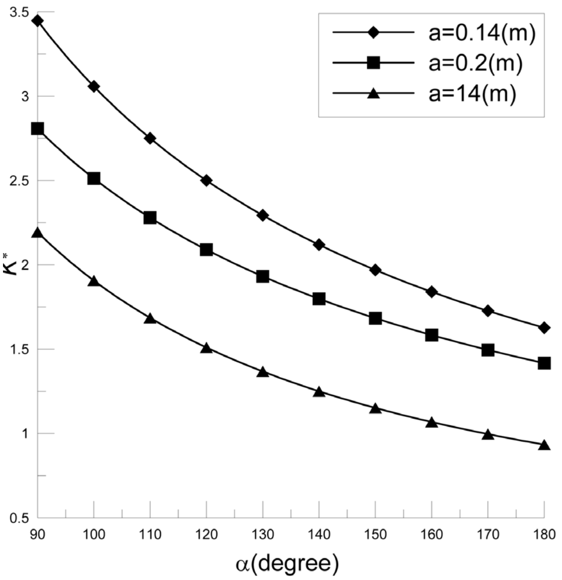


Fig. 4. Variations of normalized stress and electrical displacement intensity factors with half wedge angle at different finite radius a ($h_1 = 0.08$m, $h_2 = 0.14$m).

4. Conclusions

The antiplane electro-mechanical fields of a piezoelectric finite wedge under a pair of concentrated forces and free charges have been obtained analytically. The results show that the stresses and electrical displacements with or without singularities are continuous. In addition, the generalized stress and electrical displacement intensity factors for finite a or $a \to \infty$ have been derived. The results of the case when $a \to \infty$ are compared well with those of previous studies.

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