Applications of planar isotropic yield criteria to porous sheet metal under a deep drawing simulation

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Abstract

Macroscopic yield criteria for porous ductile sheet metals with planar isotropic matrices were proposed recently. The matrix surrounding voids is assumed to follow various planar isotropic yield functions in the current study. Effects of the macroscopic porous yield criterion and the yield function of the corresponding matrix on the sheet metal under a cylindrical cup deep drawing operation are investigated. Simulation results show that the macroscopic yield criterion of the porous sheet metal and the yield function of the corresponding matrix control the strain distribution and the strain localization. Early localization could be induced for the sheet with relatively small initial void volume fraction.

Keywords: Porous sheet metal; Planar isotropic yield criterion; Gurson yield criterion; Void volume fraction; Deep drawing forming; Finite element analysis

1. Introduction

Structural metals usually contain some micro-voids and/or second-phase particles. These imperfections generally provide strain concentration sites for void nucleation, growth, and coalescence leading to failure. Gurson [1] proposed a closed-form yield criterion to account for porous materials with isotropic matrices. Tvergaard [2] further introduced three fitting parameters into the Gurson yield criterion to obtain reasonable results of shear band instability for porous materials. The yielding of a cubic array of spherical voids under shearing conditions might occur at a lower stress than that predicted by the Gurson or the Tvergaard yield criteria, Richmond and Smelser [3] therefore introduced a concept of the effective void volume fraction into the Gurson yield criterion to account for such an overestimation.

The original Gurson yield criterion is valid only for porous materials with isotropic matrices obeying von Mises yield function. However, sheet metals usually display not only normal anisotropy but also planar anisotropy. Liao et al. [4] derived a closed-form macroscopic yield criterion for a voided sheet model with normal anisotropy but planar isotropy. Chien et al. [5] later showed that the macroscopic yield loci and the macroscopic stress–strain relationships based on the finite element results are in reasonable agreement with those based on the planar isotropic yield criterion of Liao et al. [4] after employing three additional fitting parameters as in Tvergaard [2].

Yoon et al. [6] proposed an asymmetric non-quadratic yield function (designated as YLD2000–2D hereafter) especially for aluminum alloys. They showed that the yield surface shapes with rounded vertices near the equal biaxial loading direction based on YLD2000–2D agree well with those based on the polycrystal model. Here the author selects the Hill quadratic yield function [7] and YLD2000–2D to investigate the effects of the yield function of the matrix on the strain distributions under a cylindrical cup drawing process.

2. Macroscopic planar isotropic yield criterion

Chien et al. [5] proposed the macroscopic planar isotropic yield criterion for porous materials with spherical voids as:

$$\boldsymbol{\Psi}_T = \left(\frac{\Sigma_e}{\bar{\sigma}}\right)^2 + 2q_1 f^* \cosh\left(q_2\sqrt{\frac{1+2R}{6(1+R)}}\frac{3\Sigma_m}{\bar{\sigma}}\right) - 1 - q_3(f^*)^2 = 0 \qquad (1)$$

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where, Σ_e, defined later, represents the macroscopic effective stress, $\bar{\sigma}$ is the tensile flow strength of the matrix, f^* is a function of the void volume fraction, R is the anisotropy parameter, q_i ($i = 1.3$) are fitting parameters, and Σ_m is the macroscopic mean stress. Moreover, as in Richmond and Smelser [3], the other possible macroscopic planar anisotropic yield criterion with three fitting parameters is also presented here for comparison as:

$$\boldsymbol{\Psi}_R = \left(\frac{\Sigma_e}{\bar{\sigma}}\right)^2 + 2q_1(f^*)^m \cosh\left(q_2 m\sqrt{\frac{1+2R}{6(1+R)}}\frac{3\Sigma_m}{\bar{\sigma}}\right) - 1 - q_3(f^*)^{2m} = 0 \qquad (2)$$

where a material constant m is between 2/3 and 1. For convenience of the presentation, the macroscopic yield criterion of Eq. (1) with the matrix based on YLD2000–2D, and the macroscopic yield criterion of Eq. (2) with the matrix based on the Hill quadratic yield function and YLD2000–2D are designated as TY-model, RH-model, and RY-model hereafter, respectively.

Liao [8] conducted a finite element analysis to investigate effects of planar anisotropy on porous aluminum sheet metals under plane stress conditions. He found that the macroscopic yield loci based on the RY-model with appropriate fitting parameters agrees relatively well with those based on numerical results. These three fitting parameters employed in Liao [8] are adopted for the RY-model and RH-model here even though only planar isotropy is examined in the current study. Moreover, three fitting parameters for the TY-model are chosen according to the macroscopic yield loci close to the corresponding ones based on the RY-model.

The macroscopic effective stress Σ_e in Eqs. (1) and (2) of the matrix, followed by the Hill quadratic yield function and YLD2000–2D under plane stress conditions without shear stresses due to the axisymmetric deformation, can be found in Hill [7] and Yoon et al. [6], respectively. Figure 1 shows yield loci with $R = 0.8$ based on the Hill quadratic yield function and YLD2000–2D on $\Sigma_{11}/\sigma_0 - \Sigma_{22}/\sigma_0$ plane. The rounded vertex near the equal biaxial loading direction can be clearly observed on the yield locus based on YLD2000–2D.

3. Constitutive relations

Since the author concentrates on the constitutive relation for sheet metal forming simulations, the sheet is subjected to plane stress loading conditions. The matrix material is assumed to be elastic-plastic, incompressible, and rate-insensitive. Constitutive relations between the stress rates and the strain rates are available as detailed in Liao [9].

4. Simulation results

A cylindrical cup deep drawing operation and detailed geometric parameters of the deep drawing operation can be found in Liao [9]. Figure 2 shows the meridian and

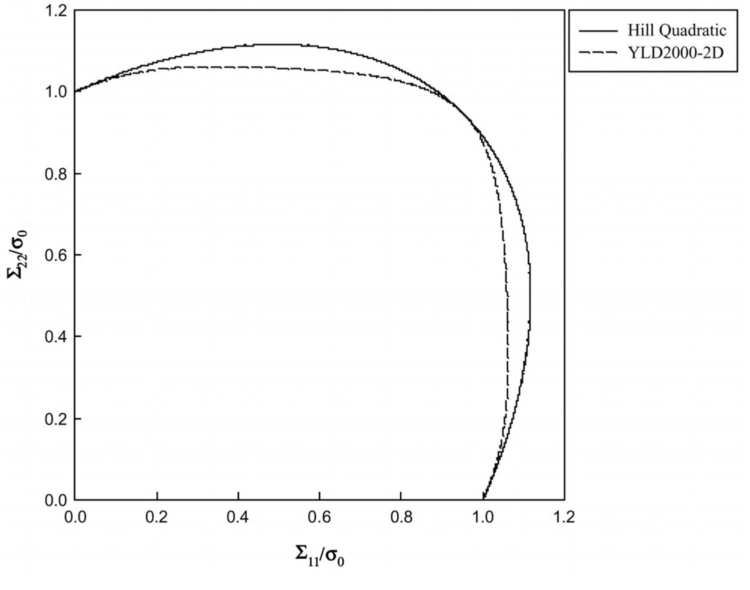

Fig. 1. The yield loci with $R = 0.8$ plotted on $\Sigma_{11}/\sigma_0 - \Sigma_{22}/\sigma_0$ plane based on the Hill quadratic yield function and YLD2000–2D.

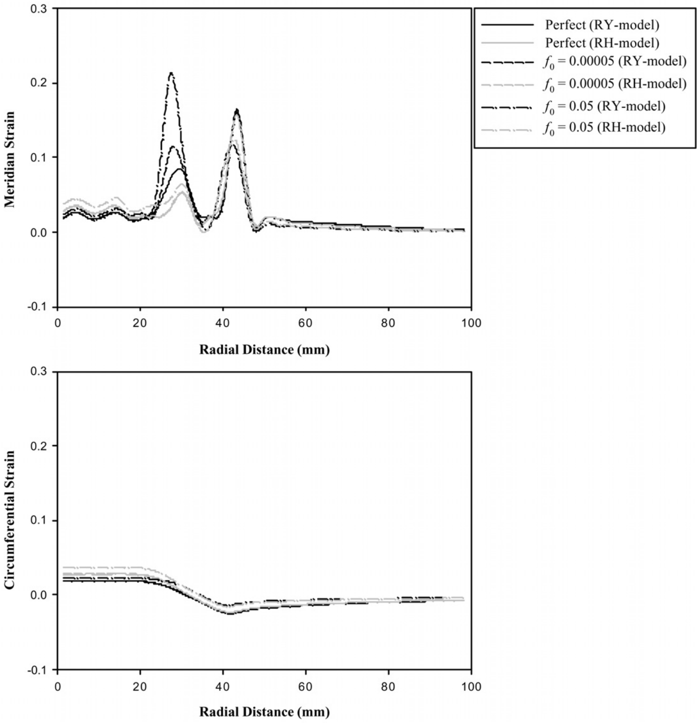

Fig. 2. Comparison of the meridian and circumferential strain distributions based on the RY-model and RH-model with various initial void volume fractions at $d = 13.9$ mm.

circumferential strain distributions based on the RY-model and RH-model with various initial void volume fractions at the punch travel distance $d = 13.9$ mm. As expected, both levels of the meridian and circumferential stain distributions rise with an increase initial void volume fraction. There are two peaks near the current positions of the punch and the die corners due to the complex loading paths such as bending, unbending, and stretching operations. The positions of the two peaks based on the RY-model are denoted as A and B. The simulation results based on two models have the same peak positions and trends of the strain distribution. An element with the largest strain in position A is chosen as a critical element here. Figure 3 shows the meridian and circumferential strain distributions based on the RY-model and TY-model at the punch travel distance $d = 12.0$ mm. Significant peak strains based on TY-model with $f_0 = 0.05$ are displayed in Fig. 3. It is obvious that the yield function of the matrix and the macroscopic porous yield criterion play an important role on the strain distribution.

Figure 4 shows the stress path for the bottom surface of the critical element based on the RY-model with $f_0 = 0.00005$, while Fig. 5 shows the corresponding void volume fractions as functions of the punch travel distance. The loading condition for the critical element is changed from the near-equal biaxial tension to the plane strain tension before the elastic unloading as shown in Fig. 4. Also the rapid increase of the void volume fraction at large punch travel distances is shown in Fig. 5.

5. Conclusions and discussion

Both effects of the macroscopic yield criterion of the porous sheet metal and the yield function of the corresponding matrix with normal anisotropy and planar isotropy on the strain distributions are significant. Sheet metals with relatively small initial void volume fraction could cause early localization under the cylindrical cup deep drawing operation.

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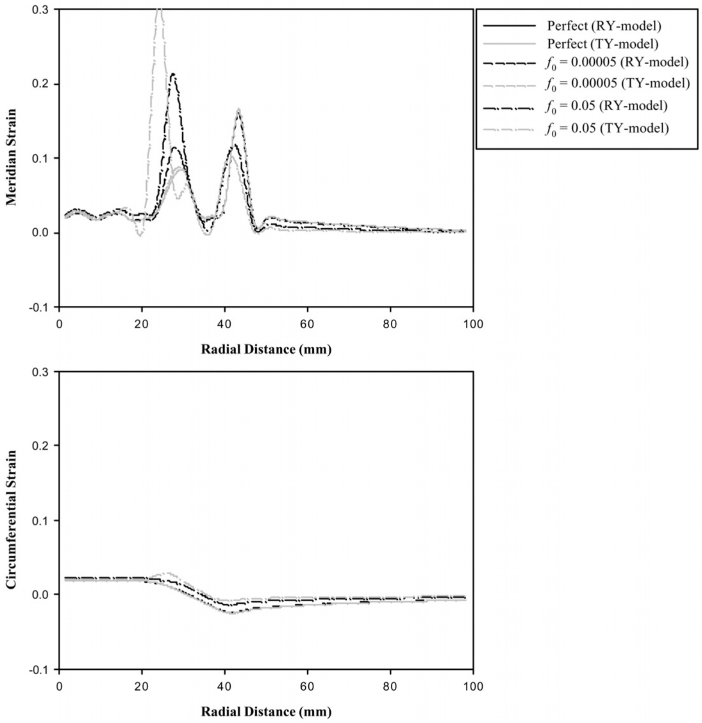

Fig. 3. Comparison of the meridian and circumferential strain distributions based on the RY-model and TY-model with various initial void volume fractions at $d = 12.0$ mm.

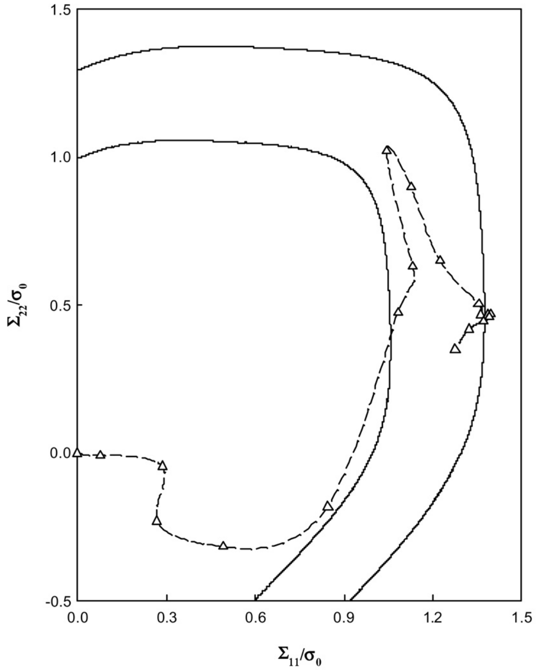

Fig. 4. The stress path for the bottom surface of the critical element based on the RY-model with $f_0 = 0.00005$ as a function of the punch travel distance.

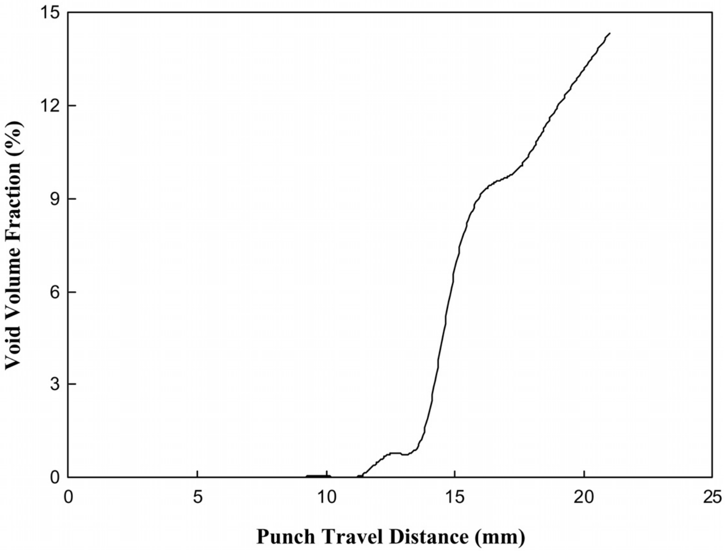

Fig. 5. The void volume fractions for the bottom surface of the critical element based on the RY-model with $f_0 = 0.00005$ as a function of the punch travel distance.

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