Numerical simulation on propagation of singularities through edges in thin hyperbolic shells

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Abstract

This short paper deals with refraction phenomenon of singularities in thin shells with edges. We clearly exhibit refraction phenomenon of singularities crossing an edge formed by two shells whose middle surfaces are hyperbolic. It appears that singularities cross the edge and propagate along characteristic curves without loss of regularity.

Keywords: Refraction; Singularity; Shells

1. Introduction

We consider the general framework of thin shell theory in the Kirchhoff Love formulation. The middle surface (with the kinematic boundary conditions) is geometrically rigid ('inhibited case' in the terminology of [1]). The limit behavior as the relative thickness ε tends to zero is mainly described by the membrane system, which is of the total order 4. The characteristics of this system are the asymptotic curves of the surface, counted two times for hyperbolic surfaces. The solutions enjoy classical properties of propagation of singularities along the characteristics. Moreover, because of the peculiarities of the system, these singularities are very sharp: for instance, a first kind discontinuity of the normal loading f^3 along a characteristic implies singularities of the normal displacement u^3 which are described by distributions (δ, δ', δ'' ..., according to the nature of the surface) along the characteristic. For small values of ϵ these singularities become thin layers of sharp variation of the solutions. It is also known [2,3] that these singularities do not satisfy classical properties of the reflection at the boundaries. Instead of this, they lose an order of intensity (or equivalently the regularity is improved by an order of magnitude); they are pseudo-reflexions [4,5,6,7]. The aim of this paper is to present numerical experiment for singularities arriving at an edge between two adjacent portions of hyperbolic middle surface in order to have a numerical evidence of the corresponding 'refraction phenomenon'. It appears that, at least in the example which is described hereafter, a singularity arriving at an edge propagates across it along the characteristics of the other portion of the surface without loss of the intensity (or without improving the regularity). We consider the case of a hyperbolic surface. Concerning the modeling of the edge, we merely prescribe that the displacement vector takes equal values on both sides of the edge. This amounts to consider the 'soft limit' in the modeling [8,9] which consider an energy form on the edge accounting for some kind of rigidity of the junction itself. Likely this point is not relevant in the limit as $\varepsilon \searrow 0$, and appears in the local structure of the layer along the edge. All along this paper, a surface will be defined by a map (Ω, Ψ), where Ω is a domain of $\mathbb{R}^2$, and Ψ denotes the position vector. All numerical results presented in this paper have been obtained by the reduced Hermite finite element.

2. Variational formulation

In this section, we give the new expression of the bilinear and linear forms which appear in the variational formulations of elasticity problems for two shells sharing a common edge. The bilinear form a[.,.] which represents the addition of the strain energy of both shells.

$$a[(u^-, u^+);\ (v^-, v^+)] = a^\varepsilon(u^-, v^-) + a^\varepsilon(u^+, v^+) \tag{1}$$

where each bilinear form $a^\epsilon(u^-, v^-)$ $a^\epsilon(u^+, v^+)$ are respectively given by:

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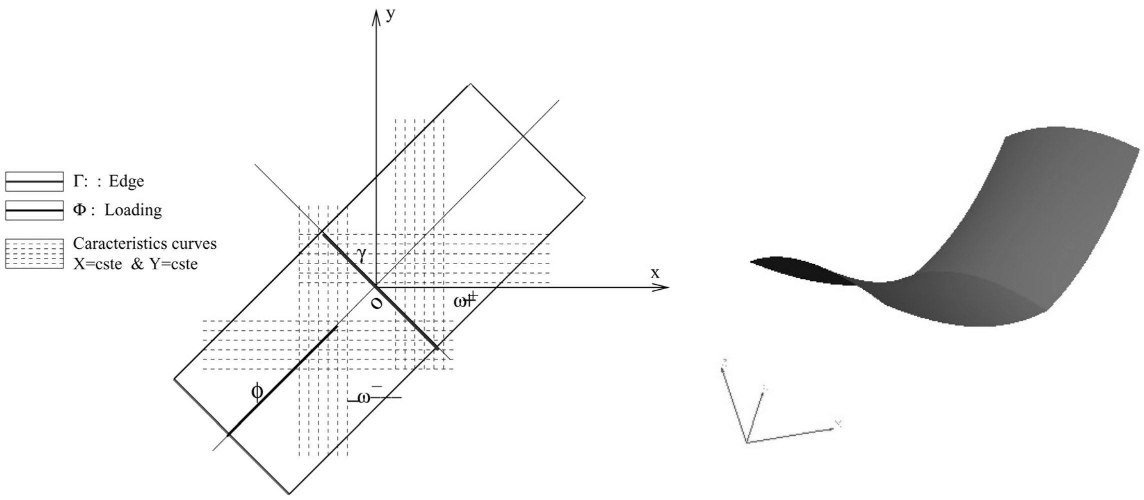

Fig. 1. Reference domain and the mapping of the middle surface in the case of two adajcent hyperbolic shells.

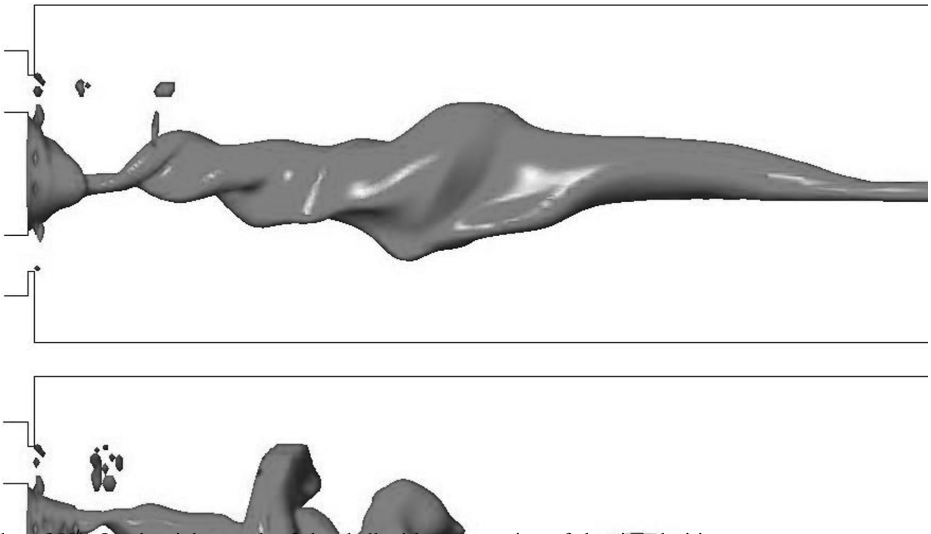

Fig. 2. Plot of U_3^ε. On the right graph of the shell with propagation of the singularities.

$$\begin{cases} a^\varepsilon(u^-,v^-) = \int_{\Omega^-} [A_-^{\alpha\beta\lambda\nu}\gamma_{\alpha\beta}(u^-)\gamma_{\lambda\nu}(v^-) + \frac{\varepsilon^2}{12}A_-^{\alpha\beta\lambda\nu}\rho_{\alpha\beta}(u^-)\rho_{\lambda\nu}(v^-)]\,dx \\ a^\varepsilon(u^+,v^+) = \int_{\Omega^+} [A_+^{\alpha\beta\lambda\nu}\gamma_{\alpha\beta}(u^+)\gamma_{\lambda\nu}(v^+) + \frac{\varepsilon^2}{12}A_+^{\alpha\beta\lambda\nu}\rho_{\alpha\beta}(u^+)\rho_{\lambda\nu}(v^+)]\,dx \end{cases} \tag{2}$$

and where the coefficients $A_-^{\alpha\beta\lambda\nu}$, $A_+^{\alpha\beta\lambda\nu}$ depend on the mechanical characteristics of the shell and on the first, second and third partial derivatives of the mapping Ψ^-: $\Omega \rightarrow S^-$ (resp. Ψ^+: $\Omega \rightarrow S^+$) which maps Ω^- (resp. Ω^+) onto the middle surface S^- (resp. S^+). $\gamma\alpha\beta$ and $\rho\alpha\beta$ are the variations of the coefficients of the first and second fundamental forms produced by $\vec{u}$.

The linear form L[.] which represents the work of the external loads can be written:

$$L[(v^-,v^+)] = \int_{\Omega^-} f^- v^- dx + \int_{\Omega^+} f^+ v^+ dx \tag{3}$$

3. Numerical results: case of two adjacent hyperbolic shells

We consider the reference domaine $\Omega = [-1,0]\,[-\frac{1}{2},\frac{1}{2}] \cup [0,1]\,[-\frac{1}{2},\frac{1}{2}]$ (see Fig. 1). The meshes of the domain Ω is generated using Modulef code.

The material is isotropic and homogeneous, with Young modulus 28,500 Nm^{-}2 and Poisson ratio 0.4. The thickness ε is equal to 10^{-4}. The shell is clamped

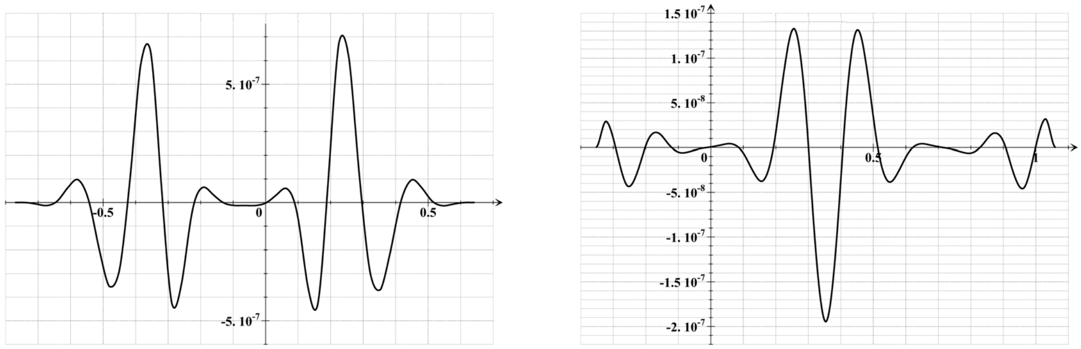

Fig. 3. Plot of cross section of U_3^h (x, y) before and after crossing the edge.

along the boundaries. The loading is $\mathbf{f} = (0, 0, f^3)$ such that:

$$f_3 = \begin{cases} \frac{1}{\text{meas}\Phi} & \text{on curve}\Phi \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

Clearly, the loading is a numerical approximation of the δ function.

On Fig. 2, with loading (4) singularities propagate along characteristics. Here characteristics are double since the shell is hyperbolic. The transmission conditions translated on the edge are that the displacement vector takes equal value on both sides of the edge.

Figure 3 represents, on the left and right respectively, a cross-section of U_h^3 before and after crossing the edge. As one can see in Fig. 3, with the considered loading (4), the range of the normal component of the displacement is very important inside the internal layers parallel to the characteristic. Due to the transmission conditions on the edge this phenomenon also appears on the other side of the shell. As it can be seen, the singualities that crossed the edge follow the characteristics without loss of magnitude.

4. Conclusion

We have built and coded an algorithm that can take into account the edge phenomenon for the hyperbolic and parabolic case. The main result is that the transmission conditions do not affect the degree of the singularities of solution. We considered several cases of parabolic and hyperbolic surfaces and always got the same behavior. Through this example and according to the nature of the geometry of the surfaces we exhibit the behavior of the singularities when they cross edges. It is clear that the degree of the singularity is preserved.

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