Modeling response of unsaturated silty sand in three-invariant stress space

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Abstract

An implicit integration algorithm has been developed to simulate stress–strain response of unsaturated soil under controlled-suction multiaxial stress states. The algorithm supports numerical analyses in the π-plane using a mixed-control constitutive driver, along with a generalized cam–clay model, incorporating a third stress invariant, Lode angle θ, within a constant-suction scheme. True triaxial data ($\sigma_1 \geq \sigma_2 \geq \sigma_3$) from constant- suction triaxial extension (TE), triaxial compression (TC), and simple shear (SS) tests, conducted on cubical specimens of silty sand, were used for tuning of the developed algorithm. The Willam–Warnke surface was adapted to the unsaturated case for simulation of soil response in three-invariant stress space (p: q: θ). Agreement between the observed and predicted soil responses highlight the potential of the developed algorithm for analyses of boundary-value problems involving unsaturated soil deposits.

Keywords: Unsaturated soil; Matric suction; Constitutive modeling; Axis translation; True triaxial testing; Implicit integration; Deviatoric plane

1. Introduction

The adoption of matric suction, $s = (u_a - u_w)$, and the excess of total stress over air pressure, $(\sigma - u_a)$, as the relevant stress-state variables has facilitated the analysis of various features of unsaturated soil behavior under axisymmetric stress states [1]. In nature, however, soils are often subjected to three-dimensional stress gradients $(\sigma_{ij} - u_a\delta_{ij})$ and $(u_a - u_w)\delta_{ij}$. Therefore, in boundary-value problems involving unsaturated soil deposits, accurate predictions of the stress–strain response of the soil-structure system requires that the soil constitutive relations be valid for all stress paths that are likely to be experienced in the field. It is in this context where a controlled-suction (c-s) true triaxial testing device plays a fundamental role in the complete stress–strain–strength characterization of unsaturated soils.

2. Objectives and scope

In this work, an implicit integration algorithm has been implemented to predict soil response along c-s multiaxial stress paths that are not achievable in a cylindrical apparatus. The developed algorithm is based on a few modifications made to the constitutive framework originally postulated by Alonso et al. [2], hereafter referred to as the Barcelona model. The implicit algorithm supports numerical analyses in the π-plane using a mixed-control constitutive driver that also incorporates the influence of Lode angle θ within a c-s scheme. The well-known Willam–Warnke surface [3], along with the Barcelona framework, was used for simulation of soil response in three-invariant stress space (p : q : θ).

True triaxial data ($\sigma_1 \geq \sigma_2 \geq \sigma_3$) from a series of c-s TC, TE, and SS tests, conducted on 4-in-per-side cubical specimens of silty sand, were used for tuning and validation of the developed algorithm. The experimental program was carried out in a cubical test cell suitable for testing soils under c-s conditions via axis-translation [4].

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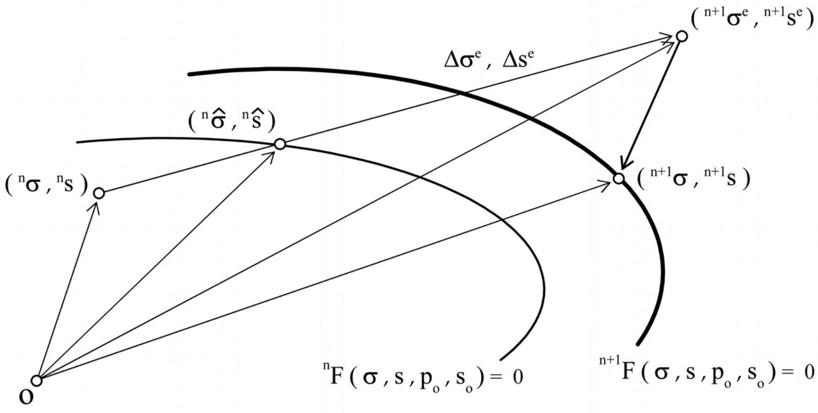

Fig. 1. Implicit CPPM-based integration scheme.

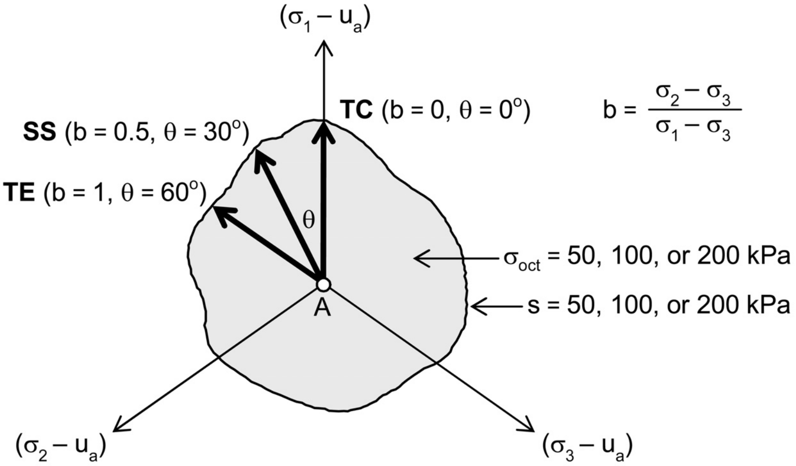

Fig. 2. Controlled-suction true triaxial testing scheme.

3. Barcelona model parameters

A detailed description of the Barcelona framework is presented in [2]. In this work, model parameters used for numerical predictions were obtained from a series of c-s isotropic loading and axisymmetric shearing tests on silty sand [4] and are summarized as follows: $\lambda(0) = 0.22$, $k = 0.011$, $\beta = 17.87\,\mathrm{MPa}^{-1}$, $r = 0.21$, $p^c = 0.036\,\mathrm{MPa}$, $G = 8.8\,\mathrm{MPa}$, $M = 0.982$, $\mathrm{k} = 1.324$, and $p_o(0) = 0.041\,\mathrm{MPa}$.

4. Implicit integration algorithm

The computational implicit integration procedure was developed as a Backward Euler (BE) return rule-based scheme for integrating the constitutive relations postulated by the Barcelona model. A solution for the unsaturated problem can be devised as the projection (closest-point-projection method-(CPPM) based) of a trial stress state ($\boldsymbol{\sigma}$, s) on to an updated yield surface ^{n+1}F, as depicted in Fig.1, where $\boldsymbol{\sigma}$ = total stress tensor, s = suction, p_o = net yield stress, and s_o = maximum past suction. A mixed-control constitutive driver was implemented as an intermediate user-model operator. Description of the calibration and validation of this implicit algorithm for axisymmetric case ($\theta = 0$) is presented by Macari et al. [5].

In this work, the updated yield surface $^{n+1}F_i$ is expressed in terms of the three stress invariants p, q, and Lode angle θ. With the help of Lode angle θ, yield functions that are well-defined in a two-dimensional

(2D) space can be expanded to a three-dimensional (3D) space by means of a function $g(\theta, c)$, where c controls the shape of the yield surface in (p : q : θ) space and represents the ratio between yield stresses in extension and compression.

The function $g(\theta, c)$ proposed by Willam and Warnke [3] to characterize behavior of concrete under general stress states, was adopted. It has been used successfully to capture constitutive response of soils [6] and is defined as follows:

$$g(\theta, c) = \frac{2(1-c^2)\cos(\theta-\pi/3) - (1-2c)\sqrt{4(1-c^2)\cos^2(\theta-\pi/3)+5c^2-4c}}{4(1-c^2)\cos^2(\theta-\pi/3)+(1-2c)^2} \quad (1)$$

With the developed algorithm, the influence of Lode angle θ on unsaturated soil response in (p : q : θ) space is

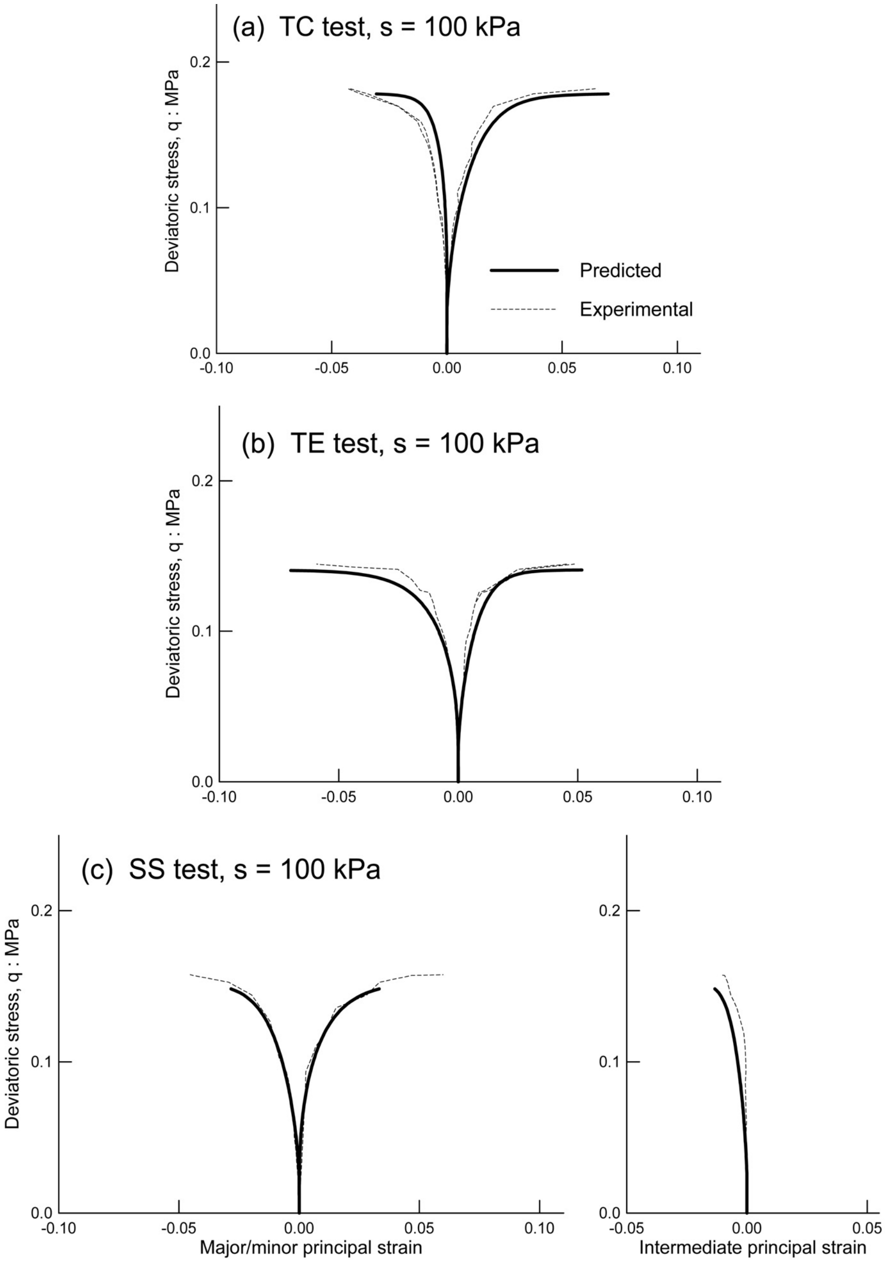

Fig. 3. Predicted and experimental deviator stress versus principal strain response.

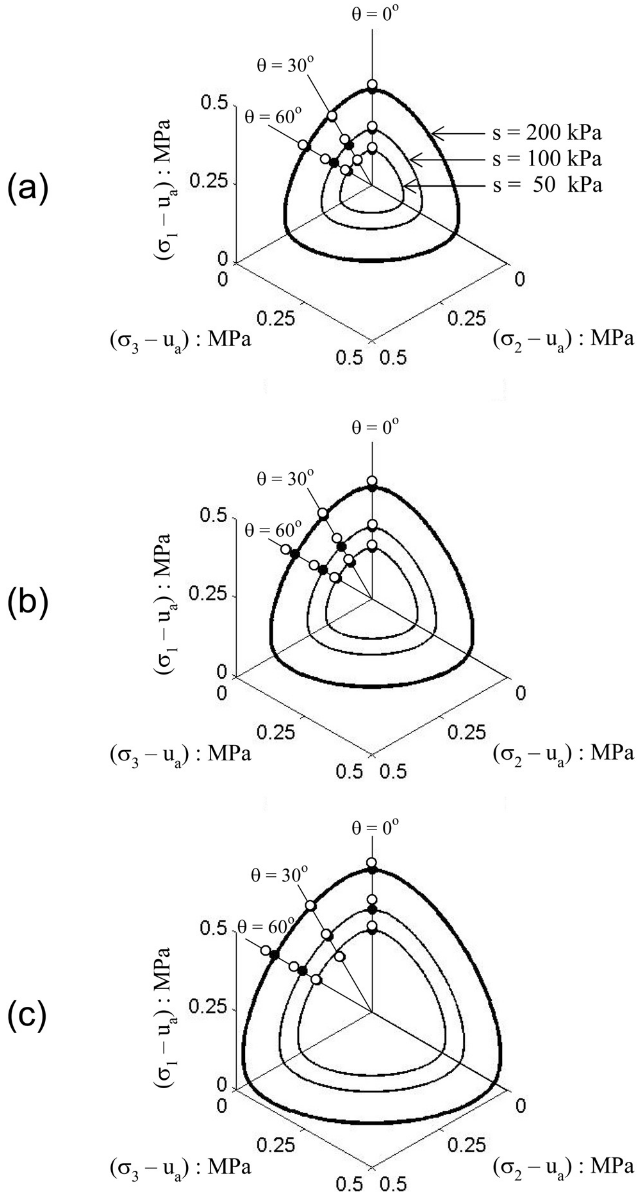

Fig. 4. Experimental (○) and predicted (●) failure loci of compacted silty sand in the π-plane.

verified against a full set of results from a series of c-s true triaxial tests on compacted silty sand.

5. Test soil and procedure

Cubical specimens of low-plasticity silty sand (SM) were compacted in place after saturation of a 5-bar disk at the bottom assembly of the cell [4]. Each specimen was subjected to a multistage testing scheme in which suction was kept constant at 50, 100, or 200 kPa. The specimen was imposed a monotonic TC, TE, or SS shearing until the deviator stress reached a peak value. At this point, the soil was brought back to initial hydrostatic condition and a new octahedral stress was applied via ramped consolidation. The same TC, TE, or SS shearing was then carried out (Fig. 2).

6. Soil response and predictions

Predictions are presented in terms of deviator stress (q) versus principal strain (ϵ) response. Net octahedral stress (σ_{oct}) and deviator stress (q) are defined as follows:

$$\sigma_{oct} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} - u_a \quad (2)$$

$$q = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \quad (3)$$

Fig. 3a shows the experimental and predicted q–ϵ responses from a TC test conducted at $\sigma_{\mathrm{oct}} = 50\,\mathrm{kPa}$ and $s = 100\,\mathrm{kPa}$. Figs 3b–c show q–ϵ responses from TE and SS tests, respectively. Predictions show reasonably good agreement with observed q–ϵ response, including the compressive (+) or expansive (−) nature of principal strains. Good agreements were also observed for TC, TE, and SS tests conducted at $\sigma_{\mathrm{oct}} = 100$ and $200\,\mathrm{kPa}$ and for $s = 50$ and $200\,\mathrm{kPa}$.

Fig. 4 shows the three-dimensional strength of silty sand in the π-plane, along with predictions of Willam–Warnke failure loci. The paramount influence of suction on the size and position of the shear strength envelopes can be seen. In general, predictions with William–Warnke function $g(\theta, c)$, defined in Eq.(1), show good agreement with observed soil response at incipient critical state.

7. Conclusions

General agreement between observed and predicted soil responses highlight the potential of the developed algorithm for analysis of boundary-value problems involving unsaturated soil deposits subjected to three-dimensional stress gradients (σ_{ij}–$u_a\delta_{ij}$) and (u_a–u_w)δ_{ij}. Further testing using an upgraded version of suction-controlled cubical test cell is currently being conducted for further refinement of the algorithm. The device is being developed under National Science Foundation Award # 0216545. This support is gratefully acknowledged.

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