Effects of unsteady aerodynamics on the dynamic response of mistuned bladed disks

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Abstract

This paper presents the preliminary results of studies on the dynamics of mistuned bladed disks including both structural and aerodynamic coupling. Aerodynamic coefficients calculated from a quasi-3D unsteady aerodynamic code are incorporated into a high-fidelity structural model employing the component mode mistuning (CMM) method. The extended CMM model is applied to the analysis of the dynamics of an industrial compressor stage. The eigenvalue structures of the tuned and mistuned systems including aerodynamic coupling are compared for the cases without structural damping, with small structural damping and with large structural damping. Also, the effect of aerodynamic coupling on the forced response of the system is investigated. For the cases studied, the large structural damping can change the eigenvalue structure of the mistuned system. Under certain conditions, the aerodynamic coupling is shown to decrease the mistuned forced response amplification factor compared to the tuned forced response.

Keywords: Mistuning; Structural coupling; Aerodynamic coupling; CMM model; Eigenvalue structure; Forced response

1. Introduction

The vibration analysis of turbomachinery rotors is important for the design and safety of commercial and military jet engines. Previous studies showed that mistuning, i.e. small differences between sectors of bladed disks, changes the dynamics of the system dramatically (e.g. [1–10]). Although aerodynamic coupling, as well as structural coupling, can have an important influence on the dynamics of bladed disks, most current studies focus on the effects of structural coupling only. Modal localization of aeroelastic modes of mistuned bladed disks has been examined [11,12]. Early mistuning studies incorporating aerodynamic coupling used mainly low fidelity structural models (e.g. [13,14]). Recently, high fidelity structural models have been developed (e.g. [15–18]). However, mistuning studies conducted using these high fidelity structural models and including aerodynamic coupling are limited. Kenyon et al. [19] discussed the aerodynamic effects on resonant blade stress of a mistuned system numerically and experimentally. Kielb et al. [20] studied the flutter of a mistuned bladed disk with both aerodynamic and structural coupling, and found weak effects of aerodynamic coupling onto the forced response.

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2. Theory

The equations of motion of the mistuned bladed disk are derived using a hybrid-interface component mode synthesis method, where the tuned bladed disk is considered as a free-interface component and the (virtual) blade mistuning is a fixed-interface component. They can be expressed as [18]

$$[(1+j\gamma)\mathbf{K}^{syn} + \mathbf{K}^{a} - \omega^2\mathbf{M}^{syn}]\mathbf{p}_{\phi}^{S} = \boldsymbol{\phi}_{\Gamma}^{S^T}\mathbf{f} \quad (1)$$

where $\boldsymbol{\phi}^{s}$ is a truncated set of normal modes of the tuned system, the subscript Γ denotes the interface between the tuned system and the virtual mistuning (i.e. the blade degrees of freedom), $\mathbf{p}_{\phi}^{S}$ are the corresponding modal coordinates, γ is the modal structural damping, and

$$\mathbf{M}^{syn} = \mathbf{I} + \boldsymbol{\phi}_{\Gamma}^{S^T}\delta\mathbf{M}\boldsymbol{\phi}_{\Gamma}^{S} \quad (2)$$

$$\mathbf{K}^{syn} = \boldsymbol{\Lambda} + \boldsymbol{\phi}_{\Gamma}^{S^T}\delta\mathbf{K}\boldsymbol{\phi}_{\Gamma}^{S} \quad (3)$$

where $\boldsymbol{\Lambda}$ is the diagonal eigenvalue matrix of the tuned system, and $\delta\mathbf{M}$ and $\delta\mathbf{K}$ denote the mistuned mass matrix and the mistuned stiffness matrix, respectively.

The modal aerodynamic coupling stiffness $\mathbf{K}^a$ can be obtained from the aerodynamic coupling stiffness matrix in the physical coordinates, $\mathbf{A}_k$, as follows:

$$\mathbf{K}^a = \boldsymbol{\phi}_\Gamma^{S^T}\mathbf{A}_k\boldsymbol{\phi}_\Gamma^S = \mathbf{q}_\phi^{CB^T}\left(\mathbf{I}\otimes\boldsymbol{\phi}^{CB^T}\right)\mathbf{A}_k\left(\mathbf{I}\otimes\boldsymbol{\phi}^{CB}\right)\mathbf{q}_\phi^{CB} \quad (4)$$

where $\otimes$ denotes a Kronecker product, $\boldsymbol{\phi}^{CB}$ is a set of cantilevered blade mode shapes, and $\mathbf{q}_\phi^{CB}$ contains the corresponding modal participation factors.

The aerodynamic coupling stiffness matrix in the cantilevered blade modal coordinates, $\tilde{\mathbf{A}}_k$, can be calculated using frequency domain unsteady CFD codes. The relationship between $\tilde{\mathbf{A}}_k$ and $\mathbf{A}_k$ can be easily obtained by the following modal transformation

$$\tilde{\mathbf{A}}_k = \left(\mathbf{I}\otimes\boldsymbol{\phi}^{CB^T}\right)(\mathbf{E}^*\otimes\mathbf{I})\mathbf{A}_k(\mathbf{E}\otimes\mathbf{I})\left(\mathbf{I}\otimes\boldsymbol{\phi}^{CB}\right) \quad (5)$$

where $\mathbf{E}$ is the complex Fourier matrix, and * denotes the conjugate transpose of a complex matrix. Finally, the modal aerodynamic coupling stiffness matrix $\mathbf{K}^a$ can be calculated as

$$\mathbf{K}^a = \mathbf{q}_\phi^{CB^T}(\mathbf{E}\otimes\mathbf{I})\tilde{\mathbf{A}}_k(\mathbf{E}^*\otimes\mathbf{I})\mathbf{q}_\phi^{CB} \quad (6)$$

When just one cantilevered blade mode shape $\boldsymbol{\varphi}^{CB}$ is considered in the aerodynamic analysis, $\tilde{\mathbf{A}}_k$ is a diagonal matrix given by

$$\tilde{\mathbf{A}}_k = diag(\tilde{A}_{k_1}, ..., \tilde{A}_{k_j}, ..., \tilde{A}_{k_N}) \quad (7)$$

where N is the number of blades.

Based on the full-potential equation using a Galerkin formulation [21], a quasi-three-dimensional model of a cascade operating in an inviscid, irrotational, isentropic flow is developed by considering the variation of stream tube heights [22].

Consider the flow between two stream surfaces. For an irrotational flow, the velocity vector can be expressed as the gradient of the scalar velocity potential $\hat{\phi}$. The conservation of mass can be expressed as

$$\frac{\partial(\hat{\rho}h)}{\partial t} + \nabla\cdot\left(\hat{\rho}\nabla\hat{\phi}h\right) = 0 \quad (8)$$

where $\hat{\rho}$ is the density of the fluid, and h is the height of the stream tube.

If the flow is isentropic, the density and the pressure are found from the integration of the momentum equation to be

$$\hat{\rho} = \rho_T\left\{1 - \frac{\gamma-1}{C_T^2}\left[\frac{1}{2}\left(\nabla\hat{\phi}\right)^2 + \frac{\partial\hat{\phi}}{\partial t}\right]\right\}^{\frac{1}{\gamma-1}} \quad (9)$$

$$\hat{p} = p_T\left\{1 - \frac{\gamma-1}{C_T^2}\left[\frac{1}{2}\left(\nabla\hat{\phi}\right)^2 + \frac{\partial\hat{\phi}}{\partial t}\right]\right\}^{\frac{\gamma}{\gamma-1}} \quad (10)$$

where ρ_T and p_T are the total density and pressure, respectively, $\hat{p}$ is the pressure, γ is the ratio of specific heats, and C_T is the total speed of sound.

Equation (8) can be transformed by using a variational principle [21,23]. Namely, the velocity potential, defined in a simple-connected domain D, which satisfies Eq. (8), renders extremum of the functional Π given by

$$\Pi = \frac{1}{T}\int_T\iint_D \hat{p}h\mathrm{d}x\mathrm{d}y\mathrm{d}t + \frac{1}{T}\int_T\oint \hat{Q}\hat{\phi}h\mathrm{d}s\mathrm{d}t \quad (11)$$

where T is the period of the unsteadiness in the flow, $\hat{Q}$ is the prescribed mass flux on the boundary, and s is the distance along the boundary.

The steady flow in each strip is calculated first. Then, the unsteady flow is linearized about the steady flow under the assumption that the unsteady flow induced by the motion of the airfoil is a small perturbation to the steady flow. In this case, the velocity potential can be expressed as the sum of a steady potential Φ and the real part of an unsteady periodic potential ϕ, i.e.

$$\hat{\phi}(x,y,t) = \Phi(x,y) + \Re[\phi(x,y)e^{j\omega t}] \quad (12)$$

with $\phi << \Phi$, $\mathrm{j} = \sqrt{-1}$ and $\Re$ representing the real part.

Using the cantilevered mode shape $\boldsymbol{\varphi}^{CB}$ as the airfoil motion, one can obtain the unsteady aerodynamic pressure p_j for a cascade of blades oscillating with the jth interblade phase angle, and $\tilde{A}_{k_j}$ can be obtained by integration over the whole blade surface area as follows:

$$\tilde{A}_{k_j} = \frac{1}{m}\int_A p_j\mathbf{n}\cdot\boldsymbol{\varphi}^{CB}\mathrm{dA} \quad (13)$$

where $\mathbf{n}$ is the local normal vector, and m is the modal mass corresponding to $\boldsymbol{\varphi}^{CB}$.

3. Results

A blisk, which is an axial compressor stage with 29 blades, is considered. First, the third group of blade-dominated modes (second flexural) is studied. The frequency of the blade alone is 9,706 Hz. The inflow relative Mach number varies from 0.359 near the hub to 0.465 near the tip. The reduced frequency based on chord is 0.93 near the tip. Figure 1 shows the eigenvalues of the tuned and mistuned bladed disks with the aerodynamic coupling for three different systems: without structural damping, with relatively small structural damping, and with large structural damping. The eigenvalues are divided by the nominal natural frequency. The damping values for the latter two systems

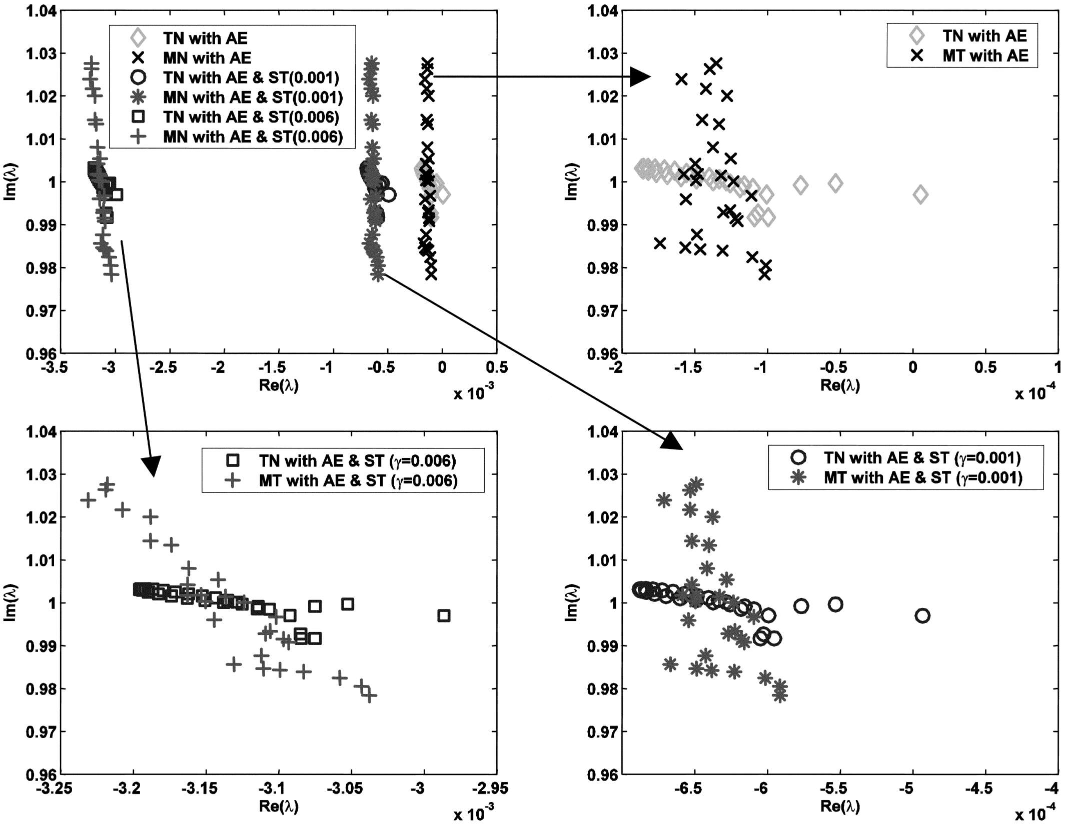

Fig. 1. Eigenvalues of the tuned and mistuned systems for the third group of blade-dominated modes with aerodynamic coupling: TN stands for tuned system, MN for mistuned system, AE for aerodynamic coupling and ST for structural coupling.

are 0.001 and 0.006, respectively. For the mistuned system the standard deviation of the blade natural frequencies is 3%. Compared to the tuned system with only aerodynamic coupling, the real parts of the eigenvalues of the mistuned system with only aerodynamic coupling narrow down, while the imaginary parts expand out. Thus mistuning makes the system more stable, and with more scattered natural frequencies. The tuned and mistuned eigenvalue structures of the system with small structural damping are similar to those of the system without structural damping, but with smaller real parts. For the system with large structural damping, the mistuned eigenvalue structure changes, and the real part of the most stable mistuned eigenvalue is less than that of the most stable tuned eigenvalue. The forced responses to an excitation with engine order 28 of the tuned and mistuned systems with small structural damping are shown in Fig. 2. The aerodynamic coupling reduces the response amplitude of the tuned and mistuned systems and the effect is small.

Next, the fourth group of blade-dominated modes (mixture of second flexural and second bending) is investigated. The frequency of the blade alone is 16,513 Hz. The inflow Mach number is 0.536 near the hub and 0.729 near the tip, and the reduced frequency based on chord is 1.246 near the tip. Figure 3 shows the tuned and mistuned eigenvalues divided by the nominal natural frequency for the different systems: without structural damping, with small structural damping, and with large structural damping. The absolute values of real parts of the eigenvalues for the system without structural damping, which stand for the aerodynamic damping, are larger than those for the third group of modes, which suggests that the effect of the aerodynamic coupling is larger for the fourth group of modes. Also, for the systems with small and large structural damping values, the tuned and mistuned eigenvalue structures do not change significantly. In this case, structural damping of 0.006 is not large enough to change the mistuned eigenvalue structure of the system. Figure 4 shows the forced responses of the system with small structural damping to an excitation with engine order 27. Surprisingly, the aerodynamic coupling increases the response of the tuned system while it decreases the response of the mistuned system, which results in a significant decrease of the amplification factor of the

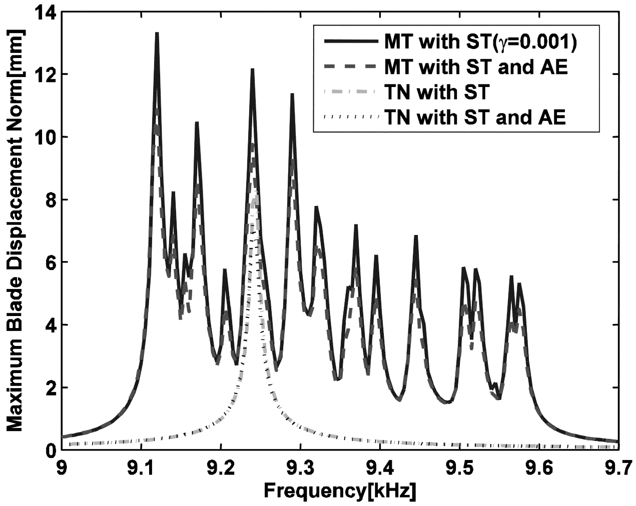

Fig. 2. Forced response of the tuned and mistuned systems with small structural damping for the third group of blade-dominated modes, with and without aerodynamic coupling.

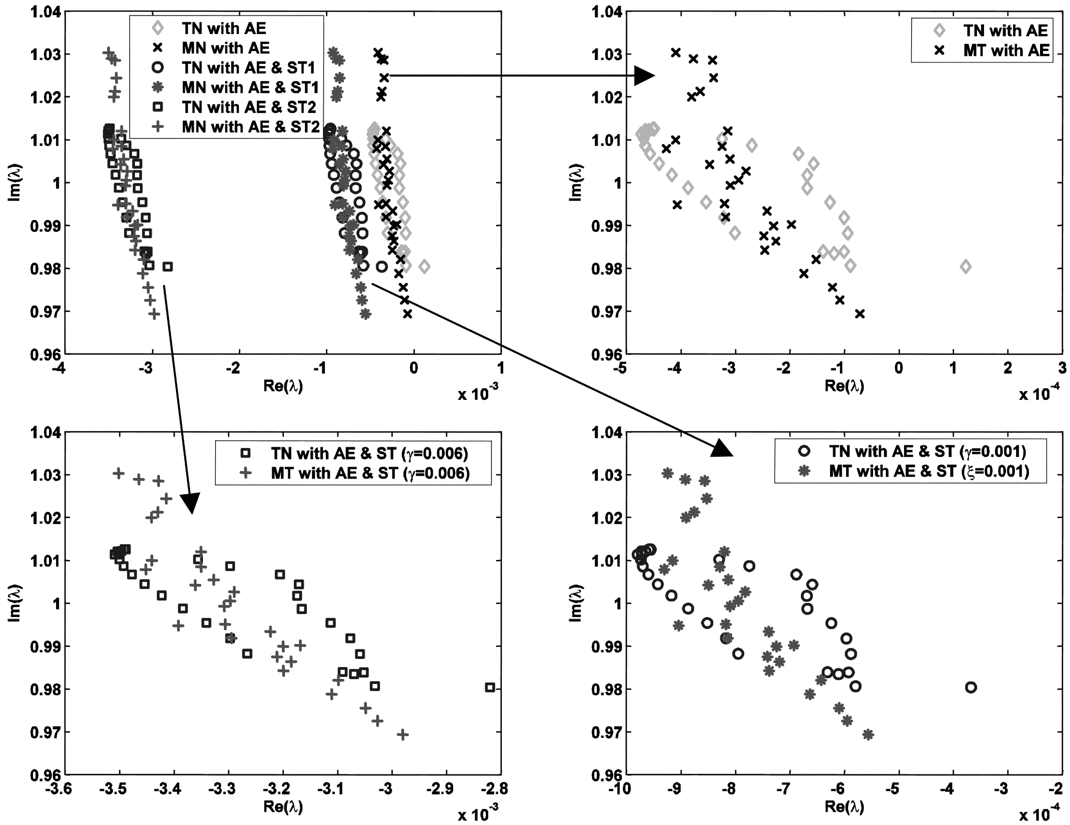

Fig. 3. Eigenvalues of the tuned and mistuned system for the fourth group of blade-dominated modes: ST1 stands for structural damping of 0.001, and ST2 stands for structural damping of 0.006.

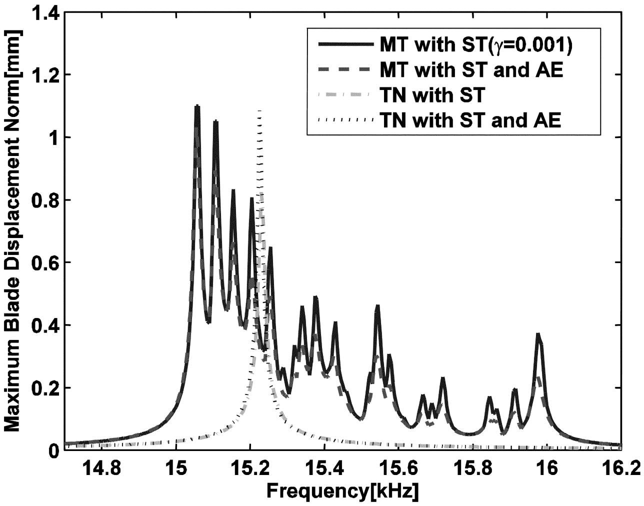

Fig. 4. Forced response of the tuned and mistuned system with small structural damping for the fourth group of blade-dominated modes.

mistuned forced response in comparison with the tuned forced response. Therefore, the aerodynamic forces can provide not only positive damping but also negative damping, which increases the forced response. The effect of negative damping is strong for a tuned system while the effect of positive damping is strong for a mistuned system.

4. Conclusions

The effects of the unsteady aerodynamic forces are introduced into a high fidelity CMM structural model to conduct the eigen-analysis and the forced response analysis of the mistuned bladed disk. The model is applied to a compressor stage with frequency mistuning. The results show that the structural damping plays a great role in the eigenvalue structure of the system. In the particular cases studied, the mistuned eigenvalue structure might be changed significantly due to the existence of relatively large structural damping. The effect of aerodynamic coupling on the forced response of the system is complicated because the aerodynamic coupling may decrease the forced response as well as increase it.

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References

[1] Tobias SA, Arnold RN. The Influence of dynamical imperfection on the vibration of rotating disk. Proc Inst Mech Eng 1957;171:669–690.
[2] Pierre C, Dowell EH. Localization of vibrations by structural irregularity. J Sound Vib 1987;114(3):549–564.
[3] Wei S-T, Pierre C. Localization phenomena in mistuned assemblies with cyclic symmetry, part I: free vibrations. J Vib Acoustics Stress Reliability Des 1988;110(4):429–438.
[4] Wei S-T, Pierre C. Localization phenomena in mistuned assemblies with cyclic symmetry, part II: forced vibrations. J Vib Acoustics Stress Reliability Des 1988; 110(4):439–449.
[5] Castanier MP, Pierre C. Lyapunov exponents and localization phenomena in multi-coupled nearly periodic systems. J Sound Vib 1995;183(3):493–515.
[6] Whitehead DS. Effect of mistuning on vibration of turbomachine blades induced by wakes. J Mech Eng Sci 1966;8(1):15–21.
[7] Whitehead DS. The maximum factor by which forced vibration of blades can increase due to mistuning. J Eng Gas Turbines Power 1998;120(1):115–119.
[8] Ewins DJ. Effects of detuning upon forced vibrations of bladed disks. J Sound Vib 1969;9(1):65–79.
[9] MacBain JC, Whaley PW. Maximum resonant response of mistuned bladed disks. ASME J Vib Acoustics Stress Reliability Des 1984;106(2):218–223.
[10] Sinha A. Calculating the statistics of forced response of a mistuned bladed disk assembly. AIAA J 1986;24(11): 1797–1801.
[11] Pierre C, Murthy DV. Aeroelastic modal characteristics of mistuned blade assemblies: mode localization and loss of eigenstructure. AIAA J 1992;30(10):2483–2496.
[12] Pierre C, Smith TE, Murthy DV. Localization of aeroelastic modes in mistuned high-energy turbines. J Propulsion Power 1994;10(3):318–328.

[13] Kaza K, Kielb RE. Flutter and response of a mistuned cascade in incompressible flow. AIAA J 1982;20(8):1120–1127.

[14] Nissim E. Optimization of cascade blade mistuning, part I: equations of motion and basic inherent properties. AIAA J 1985;23(8):1213–1222.

[15] Bladh R, Castanier MP, Pierre C. Component-mode-based reduced order modeling techniques for mistuned bladed disks, part I: theoretical models. J Eng Gas Turbines Power 2001;123(1):89–99.

[16] Bladh R, Castanier MP, Pierre C. Component-mode-based reduced order modeling techniques for mistuned bladed disks, part II: application. J Eng Gas Turbines Power 2001;123(1):100–108.

[17] Yang, M-T, Griffin JH. A reduced-order model of mistuning using a subset of nominal system modes. J Eng Gas Turbines Power 2001;123(4): 893–900.

[18] Lim S-H, Bladh R, Castanier MP, Pierre C. A compact, generalized component mode mistuning representation for modeling bladed disk vibration. In: Proc of the 44th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference and Exhibit, Norfolk, VA, 2003, AIAA paper 2003–1545.

[19] Kenyon, JA, Rabe DC, Fleeter S. Aerodynamic effects on blade vibratory stress variations. J Propulsion Power 1999;15(5):675–680.

[20] Kielb RE, Feiner DM, Griffin JH, Miyakozawa T. Probabilistic analysis of mistuned bladed disks and blisks with aerodynamic and FMM structural coupling. In: Proc. of the 9th National Turbine Engine HCF Conference, Pinehurst, NC, 2004.

[21] Hall KC. Deforming grid variational principle for unsteady small disturbance flows in cascades. AIAA J 1993;31(5):891–900.

[22] Cedar RD, Stow P. The addition of quasi-three-dimensional terms into a finite element method for transonic turbomachinery blade-to-blade flows. Int J Num Meth Fluids 1985;5(2):101–114.

[23] Bateman H. Irrotational motion of a compressible fluid. Proc Nat Acad Sci 1930;16(12):816–825.