Reliability evaluation of dynamic systems in time domain using nonlinear finite element method

A. Haldar[a,]*, J. Huh[b]

[a]University of Arizona, Department of Civil Engineering and Engineering Mechanics, Tucson, AZ 85721, USA
[b]Yosu National University, Department of Ocean System Engineering, Yeosu, 550–749, Korea

Abstract

Reliability analysis of nonlinear structures in the presence of major sources of nonlinearity is becoming an integral part of performance-based design guidelines. Finite element method is routinely used for the realistic representation of nonlinear structural behavior. However, the use of nonlinear finite element method will make the performance or limit state function, generally required for the reliability analysis, to be implicit. The authors propose a reliability evaluation technique when the limit state function is implicit. In this paper, the concept is extended to dynamic problems. The unique feature of this approach is that the uncertain dynamic loadings, including seismic loading, can be applied in time domain for the reliability evaluation.

Keywords: Reliability evaluation; Nonlinear finite element method; Implicit limit state function; Reliability evaluation in time domain

1. Introduction

Reliability evaluation of real structural systems is an essential part of performance-based design concept now being promoted worldwide. Reliability or probability of failure implies that various sources of nonlinearity are expected to be present just before failure and they must be considered appropriately to estimate the probability of failure. The finite element method is commonly used for modeling nonlinear behavior. With this approach it is straightforward to consider complicated geometric arrangements expected in a real structure, realistic connection and support conditions, various sources of nonlinearity including geometric and material, and the load path to failure. However, the deterministic finite element method fails to consider the presence of uncertainties in the load and resistance related variables and thus cannot be used for the reliability evaluation. First-order or second-order reliability methods (FORM or SORM) [1] are commonly used for the reliability analysis. In their basic form, they require that the performance or limit state functions are available in explicit form. The limit state functions in the context of the nonlinear finite element method need to be changed in each iteration, and thus will make them implicit. The author and his associates attempted to comprehensively address the subject of reliability evaluation when the limit state functions are implicit [2]. In this endeavor, they combined the desirable features of the FORM and the nonlinear finite element method, leading to the concept of the stochastic finite element method (SFEM) [2].

Several computational approaches can be used for the reliability analysis of structures with implicit limit state functions. They can be broadly divided into three categories: Monte-Carlo simulation, response surface approach, and sensitivity-based analysis. The sensitivity-based reliability approach is more elegant and in general more efficient than the simulation or response surface methods. In this approach, the sensitivity of the structural response to the input variables is computed and the information is integrated with the FORM/SORM methods to estimate the reliability. Since the limit states functions are implicit, several approximate methods can be used to compute the gradient of the performance function including the finite difference method, the classical perturbation method, and the iterative perturbation method. The iterative perturbation method is suitable for the nonlinear reliability analysis.

* Corresponding author. Tel.: +1 520 621 2142; Fax: +1 520 621 2550; E-mail: haldar@u.arizona.edu

Computational Fluid and Solid Mechanics 2005
K.J. Bathe (Editor)

Initially, the authors and their associates concentrated on static problems, i.e. the uncertain loads were applied statically to the structure. Recently, they extended the concept for dynamic problems. The unique feature of this extension is that the uncertain dynamic load can be applied in time domain. This extension is briefly discussed in this paper.

2. Nonlinear stochastic finite element method for dynamic loadings

In the nonlinear dynamic problems, not only the limit state function is implicit but it also changes with time. A response surface-based approach is expected to be appropriate; however, the basic concept of generating the response surface cannot be used since the failure region is unknown. The SFEM approach developed for the static problems can be used to locate the failure region and then response surface method can be used to generate the appropriate limit state function. The authors were successful in developing such a hybrid approach for the reliability evaluation of structures excited by short duration dynamic loadings applied in time domain by integrating the response surface method (RSM), finite element method (FEM), FORM, and an iterative linear interpolation scheme.

The major purpose of using the RSM is to generate an approximate explicit expression for a performance function. At least a second-order polynomial is necessary for the nonlinear dynamic problem. The following two types of second order polynomial are considered:

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2 \tag{1}$$

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2 + \sum_{i=1}^{k-1} \sum_{j>i}^{k} b_{ij} X_i X_j \tag{2}$$

where X_i ($i = 1, 2, \ldots, k$) is the i^{th} random variable, and b_0, b_i, b_{ii}, and b_{ij} are coefficients of the second-order polynomial. The selection of sampling points is a crucial factor in establishing the efficiency and accuracy of RSM. Saturated design (SD) and central composite design (CCD) are the two most promising techniques for generating the response surface. CCD approach is more accurate than the SD approach but is less efficient. The proposed procedure is iterative in nature. Thus, less accurate and more efficient schemes can be used in the intermediate iterations and more accurate and less efficient schemes can be used in the final iteration to increase the efficiency without compromising the accuracy. Huh and Haldar [3] consider numerous schemes. They comment that the SD using a second-order polynomial without cross terms Eq. (1) for the intermediate iterations and CCD with a full second-order polynomial Eq. (2) for the final iteration was most promising considering both efficiency and accuracy.

Since the proposed algorithm is iterative, it is necessary to improve on the selection of the location of the center point around which the sampling points are generated in subsequent iterations. Bucher and Bourgund [4] suggest an iterative linear interpolation scheme and it is used in this study. It can be mathematically represented as:

$$\mathbf{x}_{C_2} = \mathbf{x}_{C_1} + (\mathbf{x}_{D_1} - \mathbf{x}_{C_1}) \frac{g(\mathbf{x}_{C_1})}{g(\mathbf{x}_{C_1}) - g(\mathbf{x}_{D_1})} \quad \textit{if}\, g(\mathbf{x}_{D_1}) \geq g(\mathbf{x}_{C_1}) \tag{3}$$

$$\mathbf{x}_{C_2} = \mathbf{x}_{D_1} + (\mathbf{x}_{C_1} - \mathbf{x}_{D_1}) \frac{g(\mathbf{x}_{D_1})}{g(\mathbf{x}_{D_1}) - g(\mathbf{x}_{C_1})} \quad \textit{if}\, g(\mathbf{x}_{D_1}) < g(\mathbf{x}_{C_1}) \tag{4}$$

where $\mathbf{x}_{C_1}$ and $\mathbf{x}_{D_1}$ are the coordinates of the center point and the checking point for the first iteration, and $g(\mathbf{x}_{C_1})$ and $g\ (\mathbf{x}_{D_1})$ are the actual responses of the limit state function estimated from dynamic finite element analysis at $\mathbf{x}_{C_1}$ and $\mathbf{x}_{D_1}$, respectively. The point $\mathbf{x}_{C_2}$ can be used as a new center point for the next iteration. This iteration scheme needs to be continued until a preselected convergence criterion is satisfied. The conversion criterion used in the study is $(\mathbf{x}_{C_{i+1}} - \mathbf{x}_{C_i})/\mathbf{x}_{C_i} \leq |0.05|)$.

The most rigorous seismic analysis and design require that the seismic loading be applied in the time domain. However, no guideline is available on how to consider the uncertainties in both the amplitude and frequency content of the seismic loading. The uncertainty in the amplitude of the earthquake is considered by treating it as a random variable in this study. A parameter is introduced to incorporate the uncertainty in the amplitude. The uncertainty in the frequency content of an earthquake is considered indirectly. The large number of time histories recorded in close proximity of each other during a specific earthquake can be used for this purpose. They have different frequency content and the estimated reliability of a given structure will indicate the effect of uncertainty in the frequency content. The uncertainty in the frequency content can also be simulated but it is beyond the scope of this paper.

The solution strategy for the proposed algorithm can be stated as follows. The initial center point is first assumed to be the mean values of the random variables for the first iteration. The responses are calculated by conducting nonlinear FEM at the experimental sampling points for the response surface model being considered. A limit state function is thus generated in terms of k basic random variables. Using the explicit expression for the limit state function and FORM, the reliability index β and the corresponding coordinates of the checking point and direction cosines for each random variable are obtained. The coordinates of the new center point are obtained by applying the linear interpolation scheme. The updating of the location of the center point continues until it converges to a predetermined tolerance level. In the final iteration, the information on the most recent center point is used to formulate the final response surface using the CCD with full second-order polynomial. FORM is then used to calculate the reliability index and the corresponding coordinates of the most probable failure point.

2.1. Reliability evaluation

Reliability is always evaluated corresponding to a limit state function. Commonly used limit state functions are: (1) strength and (2) serviceability.

Strength limit state

The strength limit state generally represents the local behavior of structural elements. Most of the elements in a structural system can be considered as beam-column, i.e. they are subjected to both axial load and bending moment at the same time. To design steel beam–column elements, the interaction equations suggested by the American Institute of Steel Construction's manual [5] must be satisfied. The corresponding strength limit state functions can be defined as:

$$g(\mathbf{X}) = 1.0 - \left[\left(\frac{P_u}{P_n}\right) + \frac{8}{9}\left(\frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}}\right)\right] \quad if\ \frac{P}{\phi P_n} \geq 0.2 \tag{5}$$

$$g(\mathbf{X}) = 1.0 - \left[\left(\frac{P_u}{2P_n}\right) + \left(\frac{M_{ux}}{M_{nx}} + \frac{M_{uy}}{M_{ny}}\right)\right] \quad if\ \frac{P}{\phi P_n} \langle 0.2 \tag{6}$$

where P_u is the required tensile/compressive strength, P_n is the nominal tensile/compressive strength, M_{ux} and M_{uy} are the required flexural strength in the X and Y axes, respectively, and M_{nx} and M_{ny} are the nominal flexural strength in the X and Y axes, respectively. P_u, M_{ux} and M_{uy} are unfactored load effects.

Serviceability limit state

The serviceability limit state may include the lateral displacement, interstory drift, or vertical deflection. The serviceability limit state can be represented as:

$$g(\mathbf{X}) = \delta_{allow} - \delta_{\max}(\mathbf{X}) \tag{7}$$

where δ_{allow} and $\delta_{max}(\mathbf{X})$ are the allowable and the maximum serviceability values.

All the variables present in the finite element representation and loadings are expected to be present in the strength and serviceability limit state equations. Some of the variables can be treated as deterministic and the remainder should be treated probabilistically. Then the hybrid method can be used to evaluate the corresponding reliability.

The method has been extended to evaluate the reliability of steel frames with partially restrained connections [6]. Partially restrained connections add a major source of energy dissipation. Reliability of laterally weak steel frames reinforced with concrete shear walls also has been evaluated to demonstrate the wide application potential of the proposed method [7]. The efficiency and accuracy of the method will be discussed further with the help of examples during the presentation.

3. Conclusion

A stochastic finite element-based method is presented for the dynamic reliability evaluation of nonlinear structures. All major sources of nonlinearity and uncertainty can be incorporated in the algorithm. The reliability can be estimated for both the static and dynamic loading cases. The dynamic loadings including seismic loading can be applied in time domain. The reliability evaluation of real structures represented by finite elements and excited by dynamic loadings applied in time domain is emphasized in this paper.

References

[1] Haldar A, Mahadevan S. Probability, Reliability and Statistical Methods in Engineering Design. New York: John Wiley & Sons, 2000.
[2] Haldar A, Mahadevan S. Reliability Assessment Using Stochastic Finite Element Analysis. New York: John Wiley & Sons, 2000.
[3] Huh J, Haldar, A. Stochastic finite element-based seismic risk evaluation for nonlinear structures. J of the Struct Eng, ASCE 2001;127(3):323–329.
[4] Bucher CG, Bourgund U. A fast and efficient response surface approach for structural reliability problems. Structural Safety 1990;7:57–66.
[5] American Institute of Steel Construction. Manual of Steel Construction – Load and Resistance Factor Design, 3rd Edn, 2001.
[6] Huh J, Haldar, A. Seismic reliability of nonlinear frames with PR connections using systematic RSM. Probabilistic Engineering Mechanics 2002;17(2):177–190.
[7] Lee SY, Haldar A. Reliability analysis of frame and shear wall structural systems – dynamic loading. J of the Struc Eng, ASCE 2003;129(2):233–240.