Plasticity with nonlocal damage, with application to concrete cracking

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Abstract

The present paper deals with modeling the strain softening behavior of concrete by means of plasticity and nonlocal damage. A stress-based local plasticity model is combined with an integral-type nonlocal damage model driven by the plastic strain. The evolution of the plastic zone is controlled by the hardening law of the local plasticity model. An exponential hardening law (decreasing hardening modulus) is used to describe the evolution of the strain profile from distributed to fully localized. The response of a bar in uniaxial tension and a compact tension test of plain concrete is modeled by the presented plastic-damage model.

Keywords: Plasticity; Damage mechanics; Nonlocal; Concrete; Failure; Localization

1. Introduction

The failure process of cohesive-frictional materials, such as concrete, rock and soils, is accompanied by localized deformation patterns, which are usually modeled by constitutive laws with strain softening, i.e. decreasing stress with increasing strain. If these constitutive laws are local, i.e. the response at a point depends only on the current values and the history at that point alone, strain softening leads within the finite element method to mesh dependent zones of inelastic strains and strongly mesh-dependent dissipation associated with the failure process.

A remedy to the pathological mesh dependence are nonlocal constitutive laws, where the response at one point depends not only on the current values and the history at that point, but also on those of the neighboring points. In these models a characteristic length is incorporated, which allows to describe the dissipation of a localized failure process independently of the mesh and, for many models, the localized deformations in terms of a continuous strain profile. For strongly nonlocal models the interaction between neighboring points is achieved by integral-type averaging. These models have been proposed in [1,2] for damage mechanics. For strain-based scalar damage models the stress evaluation approach is often explicit and, therefore, integral type nonlocal damage models are computationally efficient. Integral-type nonlocal models for stress-based plasticity, in which the yield stress depends on the nonlocal cumulative plastic strain, were developed, for instance, in [3,4]. This type of nonlocal formulation requires a complicated stress-return algorithm due to the coupling among Gauss points and the implicit stress evaluation procedure.

In the present paper, we propose an alternative technique: the softening response is modeled by a combination of local plasticity based on the effective stress and nonlocal scalar damage driven by the nonlocal cumulative plastic strain. This approach is computationally efficient, since the plasticity part with the implicit stress return remains local and only the damage part with the explicit stress evaluation procedure is made nonlocal. This approach has been studied analytically by the authors in [5] with focus on the conditions for a mesh-independent description of the zone of inelastic strains, where it has been shown that the size of the plastic zone is controlled by the values of the plastic hardening modulus H_p and the critical hardening modulus $H_{p,crit}$, determined by classical bifurcation analysis. As long as H_p is larger than $H_{p,crit}$ a continuous zone of plastic strains is guaranteed. When H_p is smaller than $H_{p,crit}$ the plastic zone is fully

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localized and therefore mesh-dependent, but the dissipation is still mesh-independent. Thus the size of the plastic zone is controlled by the difference of H_p and $H_{p,crit}$.

For concrete cracking, for instance, the fracture process is characterized by a transition from a distributed zone of inelastic strains to a localized stress-free crack. This continuous process is modeled in the present work by a hardening law of the local plasticity model with decreasing hardening modulus under increasing plastic strains. In the following sections the constitutive model is briefly presented and the capabilities of the modeling approach is demonstrated by simulation of a compact tension test of plain concrete.

2. Constitutive model

In the present section the framework of the combination of local stress-based plasticity and nonlocal strain-based scalar damage is presented. The general stress-strain relation for this type of model is

$$\boldsymbol{\sigma} = (1-\omega)\mathbf{D}_e : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p) = (1-\omega)\bar{\boldsymbol{\sigma}} \tag{1}$$

where $\boldsymbol{\sigma}$ is the stress, ω is the damage variable, $\mathbf{D}_e$ is the elastic stiffness, $\boldsymbol{\varepsilon}$ is the strain, $\boldsymbol{\varepsilon}_p$ is the plastic strain and $\bar{\boldsymbol{\sigma}}$ is the effective stress.

The plasticity model is based on the effective stress and consists of the following equations:

$$f_P(\bar{\boldsymbol{\sigma}}, \kappa_p) = \tilde{\sigma}(\bar{\boldsymbol{\sigma}}, \kappa_p) - \sigma_y(\kappa_p) \tag{2}$$

$$\dot{\boldsymbol{\varepsilon}}_p = \dot{\lambda}\frac{\partial g_p}{\partial \bar{\boldsymbol{\sigma}}}(\bar{\boldsymbol{\sigma}}, \kappa_p) \tag{3}$$

$$\dot{\kappa}_p = \dot{\lambda} k_p(\bar{\boldsymbol{\sigma}}, \kappa_p) \tag{4}$$

$$f_p \leq 0 \quad \dot{\kappa}_p \geq 0 \quad \dot{\kappa}_p f_p = 0 \tag{5}$$

where f_p is the yield function, $\tilde{\sigma}$ is the equivalent stress, κ_p is the plastic hardening parameter, $\dot{\lambda}$ is the plastic multiplier, g_p is the plastic potential, and k_p is a function that relates rates of the plastic multiplier and the hardening variable. In the present study, the Rankine criterion is applied. The damage model is based on the plastic strain and is described by the following equations:

$$f_d(\boldsymbol{\varepsilon}_p, \kappa_d) = \bar{\tilde{\varepsilon}}(\boldsymbol{\varepsilon}_p) - \kappa_d \tag{6}$$

$$\omega = g_d(\kappa_d) \tag{7}$$

$$f_d \leq 0, \quad \dot{\kappa}_d \geq 0, \quad \dot{\kappa}_d f_d = 0 \tag{8}$$

where f_d is the damage loading function, $\bar{\tilde{\varepsilon}}$ is the nonlocal equivalent strain, and κ_d is the damage hardening-softening variable. The nonlocal equivalent strain is the weighted average of the cumulative plastic strain,

$$\bar{\tilde{\varepsilon}}(\mathbf{x}) = \int_V \alpha(\mathbf{x}, \mathbf{s})\kappa_p(\mathbf{s})\, \mathrm{d}\mathbf{s} \tag{9}$$

Here, $\alpha(\mathbf{x}, \mathbf{s})$ is a nonlocal weight function that describes the strength of interaction between points $\mathbf{x}$ and $\mathbf{s}$ and decays with increasing distance between these points. The weight function is usually non-negative and normalized such that $\int_V \alpha\,(\mathbf{x}, \mathbf{s})\,\mathrm{d}\mathbf{s} = 1$ for all $\mathbf{x} \in V$. The algorithm for the stress update of this type of model is divided into two steps. First, the update of the effective stress for the plasticity model is carried out by an implicit algorithm; then the nonlocal damage part is evaluated explicitly from the plastic strain obtained in the first step.

As discussed in the introduction, the zone of the plastic strains depends on the hardening modulus of the plasticity model. In the present study, an exponential hardening law of the form

$$\sigma_y(\kappa_p) = \sigma_L + (\sigma_0 - \sigma_L)\exp(-\kappa_p/\kappa_f) \tag{10}$$

is used, where σ_0 is the initial yield stress, σ_L is the limit yield stress (larger than σ_0) and κ_f is a control parameter. The plastic hardening modulus H_p is therefore given by

$$H_p = \frac{\partial \sigma_y}{\partial \kappa_p} = -\frac{(\sigma_0 - \sigma_y)}{\kappa_f}\exp(-\kappa_p/\kappa_f) \tag{11}$$

with $\lim_{\kappa_p \to \infty} H_p = 0$. This hardening law results in a plastic zone that shrinks with increasing κ_p and finally tends to localize fully. Thus, it is expected to obtain a realistic evolution of the strain profile for tensile cracking.

3. Examples

The first structural example is a bar subjected to uniaxial tension, as depicted in Fig. 1(a). The response is presented in Figs. 1(b), 2(a) and 2(b) in terms of the load–displacement curve and graphs showing the evolution of the damage variable and the plastic strain, respectively. The load–displacement diagrams converge upon mesh refinement and the area under them (dissipated energy) remains nonzero. For comparison, the dashed curves show the load–displacement diagram for the unstable uniform solution. The plastic strain is distributed continuously, and damage is uniform in the plastic zone and decreases continuously to zero in intervals of width R around the plastic zone, which shrinks considerably.

The second example is a compact tension test of plain concrete. The geometry and the finite element mesh are shown in Fig. 3(a). The load versus the crack mouth opening (CMOD) obtained by simulation is compared to experimental results reported in [6] in Fig. 3(b). The

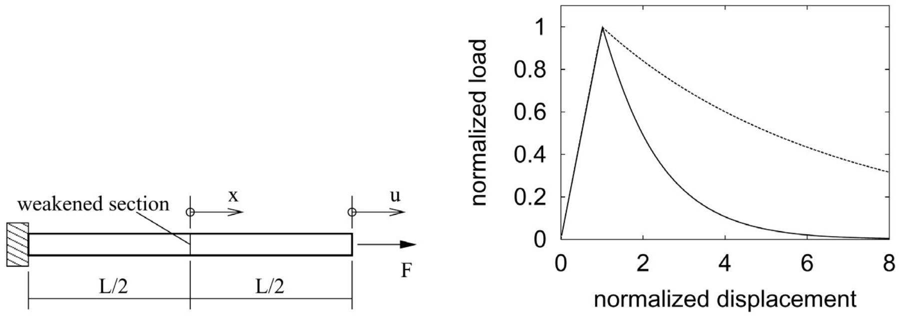

Fig. 1. (a) Geometry and loading setup, and (b) load–displacement diagram for a bar in uniaxial tension.

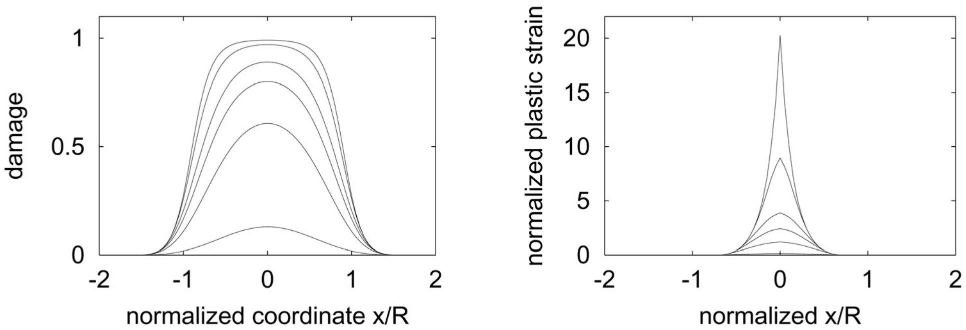

Fig. 2. Evolution of (a) the damage variable and (b) the plastic strain for a bar in uniaxial tension.

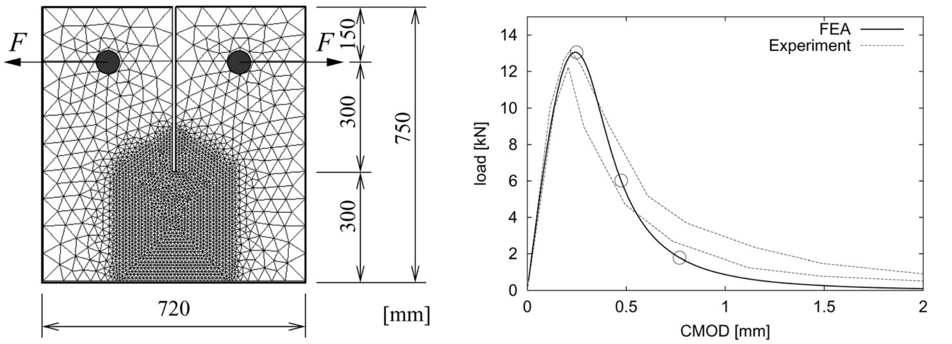

Fig. 3. (a) Geometry and finite element mesh of the compact tension test. (b) Load versus crack mouth opening obtained in the finite element analysis compared to the experimental results reported in [6].

plots of the zone of the plastic strain increment at three stages of analysis (marked in Fig. 3(b)) are depicted in Fig. 4. The plastic zone shrinks with increasing crack opening. At a late stage the zone of the plastic strain increment is almost localized in the upper part and only at the 'crack' tip the plastic strain increment is distributed in a wider zone.

4. Conclusions

In the present paper, a framework for modeling strain softening by means of plasticity and nonlocal damage has been presented. The plasticity model is local and based on the effective stress. The isotropic damage model is driven by the nonlocal cumulative plastic strain. This approach is computationally efficient since the plasticity model with the implicit stress return is local

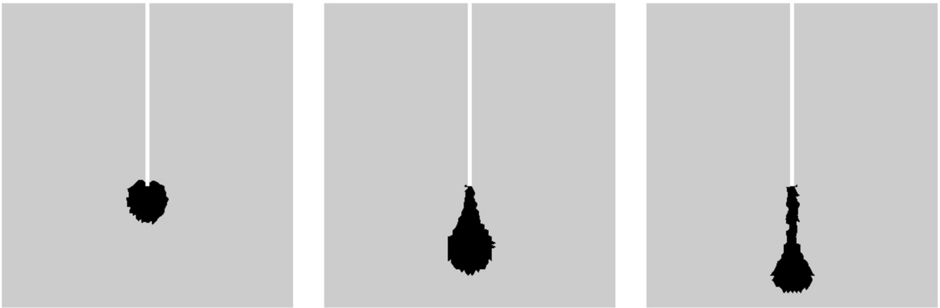

Fig. 4. The zone of the plastic strain increment for the compact tension test for three stages in the softening regime (as marked in Fig. 3(b)).

and only the explicit damage part is nonlocal. Both the strain profile and the dissipated energy are described independently of the mesh.

With an exponential hardening law (decreasing hardening modulus) a shrinking plastic zone is obtained, which allows a realistic description of the evolution of the strain profile observed in concrete cracking.

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