Evolutionary shape optimization of plates with reinforcements

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Abstract

The aim of this paper is to present the formulation and application of the coupled boundary and finite element method (BEM/FEM) and an evolutionary algorithm (EA) to optimization of plates stiffened by beams subjected to static or dynamic loads. The dual reciprocity BEM is used to model plates and the dynamic FEM formulation is used to model beams. The BEM and FEM models are combined by transforming the FEM equations into the BEM form. The problem of optimization is solved using an evolutionary algorithm. Numerical examples of optimization are shown.

Keywords: Optimization; Boundary element method; Finite element method; Coupled BEM/FEM; Reinforcement; Evolutionary algorithm

1. Introduction

Nowadays, the BEM and the FEM are the most popular computer techniques suitable for solving many engineering problems. However, when used separately, both methods have advantages and disadvantages. The advantages of both methods can be exploited by combining them. There are several ways of coupling of BEs with the FEs [1]. In the present approach, the equations for different domains are assembled and the set of equations for the whole structure is obtained using compatibility of displacements and equilibrium of tractions between the common interfaces.

Reinforced composite structures were analyzed and optimized for instance by Górski et al. [2]. The results of optimization by the EA for a simply supported stiffened plate were compared with the solutions obtained using the systematic search method showing a very good agreement. In the present work, homogeneous reinforced structures are considered. The aim of reinforcement is to provide static or dynamic stiffness and strength. The problem of analysis and optimization is solved using the coupled BEM/FEM and the EA [3], respectively. The design variables of the problem are dimensions of the structure.

2. A coupled boundary and finite element method

In the present approach, the direct coupling of the BE and FE matrices, by transforming the FE matrices into an equivalent BE-type, is performed. The FE nodal forces are transformed into the BE tractions by using a special transformation matrix [4].

Consider the body subjected to dynamic load and consisting of two different materials occupying regions Ω^1 and Ω^2 as shown in Fig. 1. The Ω^1 domain is predominant and it is modeled as a plate by the BEM. The Ω^2 domain is modeled as a beam by the FEM. The external boundary of the plate is Γ^1 and the interface connecting two materials is Γ^{12}. The numerical solution is obtained after dividing the body into the boundary and finite elements. For the Ω^1 region, the dual reciprocity BEM [5] allows the formulation of the following system of equations of motion in a matrix form:

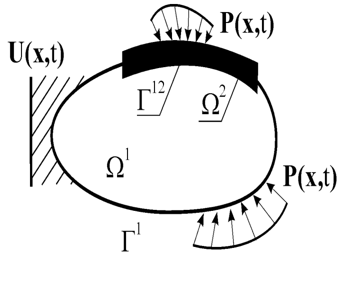

Fig. 1. A reinforced structure.

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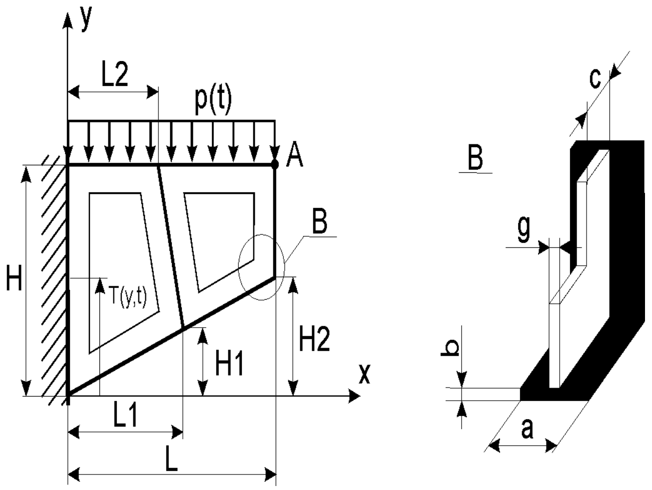

Fig. 2. Reinforced cantilever plate.

$$\begin{bmatrix} \boldsymbol{M}^1 & \boldsymbol{M}^{12} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{U}}^1 \\ \ddot{\boldsymbol{U}}^{12} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{H}^1 & \boldsymbol{H}^{12} \end{bmatrix} \begin{Bmatrix} \boldsymbol{U}^1 \\ \boldsymbol{U}^{12} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{G}^1 & \boldsymbol{G}^{12} \end{bmatrix} \begin{Bmatrix} \boldsymbol{P}^1 \\ \boldsymbol{P}^{12} \end{Bmatrix} \tag{1}$$

and for the Ω^2 region the governing FEM equations are:

$$\boldsymbol{M}^{21}\ddot{\boldsymbol{U}}^{21} + \boldsymbol{K}^{21}\boldsymbol{U}^{21} = \boldsymbol{T}^{21}\boldsymbol{P}^{21} \tag{2}$$

where $\boldsymbol{M}$ represents the mass matrices, $\boldsymbol{H}$ and $\boldsymbol{G}$ are the BEM coefficients matrices, $\boldsymbol{K}$ is the FEM stiffness matrix, and $\boldsymbol{T}$ is the matrix that expresses the relationship between the FE nodal forces and the BE tractions; $\boldsymbol{U}$, $\ddot{\boldsymbol{U}}$ and $\boldsymbol{P}$ are respectively displacement, acceleration and traction vectors. The superscripts denote the matrices that correspond to the appropriate boundaries. The displacement compatibility conditions and the traction equilibrium conditions over the interface Γ^{12} are:

$$\boldsymbol{U}^{12} = \boldsymbol{U}^{21};\ \boldsymbol{P}^{12} = -\boldsymbol{P}^{21} \tag{3}$$

Using the above conditions in Eq. (1) and (2), the whole system of equations for the structure in Fig. 1 can be rearranged as:

$$\begin{bmatrix} \boldsymbol{M}^1 & \boldsymbol{M}^{12} \\ 0 & \boldsymbol{M}^{21} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{U}}^1 \\ \ddot{\boldsymbol{U}}^{12} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{H}^1 & \boldsymbol{H}^{12} & -\boldsymbol{G}^{12} \\ 0 & \boldsymbol{K}^{21} & \boldsymbol{T}^{21} \end{bmatrix} \begin{Bmatrix} \boldsymbol{U}^1 \\ \boldsymbol{U}^{12} \\ \boldsymbol{P}^{12} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{G}^1 & 0 \\ 0 & \boldsymbol{T}^{21} \end{bmatrix} \begin{Bmatrix} \boldsymbol{P}^1 \\ \tilde{\boldsymbol{P}}^{21} \end{Bmatrix} \tag{4}$$

where $\tilde{\boldsymbol{P}}^{21}$ denotes prescribed tractions at the interface. The above system of equations is rearranged according to the boundary conditions and solved step-by-step giving the unknown displacements and tractions on the external boundary and the interface at each time step. The method can be used for the static analysis by assuming that the accelerations of all nodes are equal zero.

3. Numerical examples

The linear-elastic, isotropic and homogeneous reinforced cantilever plate shown in Fig. 2, is considered. The reinforcement is applied at the whole non-fixed outer boundary and at the interface between two BE regions (plates). The plate is subjected to static or dynamic loads. Two criteria of optimization, the same for statics and dynamics, are used: (1) maximization of stiffness of the body and (2) minimization of support tractions. The first objective function is the maximum vertical displacement u_A at point A (see Fig. 2). The second objective function is the maximum resultant traction $T(y)$ at the fixed boundary. The objective functions are minimized with respect to design variables *H1*, *H2* and *L1*, *L2*, defining dimensions of the structure.

The length and height of the structure are respectively $L = 50\,\text{cm}$ and $H = 40\,\text{cm}$. The other dimensions are: $a = 5\,\text{cm}$, $b = 1\,\text{cm}$, $c = 5\,\text{cm}$ and $g = 1\,\text{cm}$. The values of the *H1*, *H2* and *L1*, *L2* variables belong to the intervals from 0 to 25 cm and 15 to 35 cm, respectively. For the dynamic problem, the plate is subjected to the sinusoidal load $p(t) = p_o \sin(2\pi t/T)$. The amplitude of the load is $p_o = 1\,\text{MPa}$ and the period of time is $T = 5\,\text{ms}$. The time of analysis (Houbolt scheme) is 12 ms and the time step $\Delta t = 0.02\,\text{ms}$. The material of the structure is steel in plane stress, for which modulus of elasticity is $E = 210\,\text{GPa}$, Poisson's ratio $\nu = 0.3$ and density $\rho = 7860\,\text{kg/m}^3$.

Table 1
Results of optimization – maximization of stiffness

Test No.	Statics					Dynamics				
	H1	*H2* [cm]	*L1*	*L2*	U_A [10^{-5} cm]	*H1*	*H2* [cm]	*L1*	*L2*	U_A [10^{-5} cm]
1	25.00	25.00	30.58	35.00	188.01	25.00	25.00	29.74	35.00	245.46
2	25.00	25.00	30.59	35.00	188.01	25.00	25.00	29.74	35.00	245.46
3	25.00	25.00	30.59	35.00	188.01	25.00	25.00	29.74	35.00	245.46
4	25.00	25.00	30.59	35.00	188.01	25.00	25.00	29.74	35.00	245.46
5	25.00	25.00	30.62	35.00	188.01	25.00	25.00	29.74	35.00	245.46
6	11.08	25.00	15.00	35.00	188.32	25.00	25.00	29.74	35.00	245.46
7	11.09	25.00	15.00	35.00	188.32	25.00	25.00	29.74	35.00	245.46
8	11.09	25.00	15.00	35.00	188.32	25.00	25.00	29.74	35.00	245.46
9	11.10	25.00	15.00	35.00	188.32	10.02	25.00	15.00	35.00	253.16
10	11.11	25.00	15.00	35.00	188.32	10.14	25.00	15.10	35.00	253.33

Table 2
Results of optimization – minimization of support tractions

Test No.	Statics					Dynamics				
	H1	*H2* [cm]	*L1*	*L2*	*T* [MPa]	*H1*	*H2* [cm]	*L1*	*L2*	*T* [MPa]
1	10.10	25.00	15.00	35.00	15.41	10.39	25.00	15.00	34.98	19.47
2	10.11	25.00	15.00	35.00	15.41	10.39	25.00	15.00	34.98	19.47
3	10.11	25.00	15.00	35.00	15.41	10.39	25.00	15.00	34.98	19.47
4	10.11	25.00	15.00	35.00	15.41	10.39	25.00	15.00	34.98	19.47
5	10.12	25.00	15.00	35.00	15.41	10.39	25.00	15.00	34.98	19.47
6	9.73	2.78	15.00	35.00	15.42	10.39	25.00	15.00	34.98	19.47
7	9.73	2.79	15.00	35.00	15.42	10.39	25.00	15.00	34.98	19.47
8	9.74	2.75	15.00	35.00	15.42	10.52	24.76	15.00	35.00	19.47
9	9.74	2.82	15.00	35.00	15.42	11.90	23.17	15.00	34.89	19.47
10	9.75	2.77	15.00	35.00	15.42	11.90	23.17	15.00	34.89	19.47

The total number of boundary elements is 84. The total number of finite elements is 72. The quadratic elements (with 2 degrees of freedom per node) are employed for the BEM mesh. The beam elements (with 3 degrees of freedom per node) are used for the FEM mesh. The number of boundary and finite elements is constant, which simplifies significantly the modification of BE and FE discretization.

In all examples the number of chromosomes in the population is 10 and the number of generations is 200. For each example 10 tests are performed.

3.1. Maximization of stiffness

In this example the results of optimization for statics and dynamics are presented. Initially, the computations are performed for the so called reference plate, for which the values of design variables are: $H1 = 10$ cm, $H2 = 20$ cm and $L1 = L2 = 25$ cm.

The criterion of optimization is maximization of stiffness of the body. The results of optimization using the EA are presented in Table 1. Two different solutions are obtained for the static and dynamic problem. However, the values of the objective function are similar, the shape of the structure is different. The decrease of the maximal displacement for the optimal structure in comparison with the reference plate is about 48 and 56%, while the reduction of total weight is about 14 and 13%, for the static and dynamic problems respectively. The influence of the reinforcement on stiffness of the reference plates was also investigated. In this case, the reduction of the maximal static and dynamic displacement at point A is about 50%, in comparison with plates without reinforcement.

3.2. Minimization of support tractions

The criterion of optimization is minimization of tractions at the fixed boundary (see Fig. 2). The results

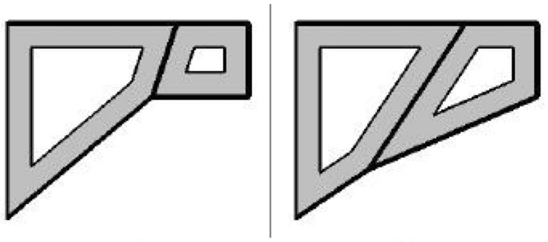

Fig. 3. Optimal plates: (a) maximization of stiffness and (b) minimization of support tractions.

of optimization using the EA are presented in Table 2. As in the previous example, for the static problem similar values of the objective functions, for different shapes of the structure, are obtained. The decrease of the maximal traction for the optimal structure is about 33 and 44%, for the static and dynamic problems respectively. The total weight has increased by 1% in both cases, in comparison with the reference plate. The influence of the reinforcement on strength of the reference plates was also investigated. In this case, the reduction of the maximal resultant static and dynamic traction at the fixed boundary is about 25%, in comparison with non-stiffened plates.

The optimal structures for the two considered criteria of optimization are shown in Fig. 3. One can observe (see Tables 1 and 2) that the shape of the optimal plates for the static and dynamic problems is very similar.

4. Conclusions

In the paper, the coupled BEM/FEM and the evolutionary algorithm are used in optimization of statically or dynamically loaded reinforced structures. The reinforcement has improved static or dynamic stiffness and strength. The decrease of maximal displacements and tractions for the optimal structures is significant, in comparison with plates before optimization.

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