A parametric study of bulging factors for unstiffened and stiffened cylindrical shells

Christian Hideki Furukawa[a], Miguel Luiz Bucalem[a,*], Carlos Eduardo Chaves[b]

[a]*Departamento de Engenharia de Estruturas e Fundações da Escola Politécnica da Universidade de São Paulo, Av. Prof. Almeida Prado, trav.2 no. 83, Cidade Universitária, São Paulo SP 05508–900, Brazil*
[b]*Instituto Tecnológico de Aeronáutica, Praça Marechal Eduardo Gomes, 50, S.J. dos Campos, São Paulo SP 12228–904, Brazil*

Abstract

Bulging factors represent the ratio of increase in stress intensity factors for a pressurized shell when compared with plate conditions. Therefore, they influence stress-intensity-based crack-propagation methodologies. Since in a real fuselage the bulging factors depend on a number of parameters such as pressure level, skin thickness, structural area of the stiffeners, and distance between stiffeners, among others, a parametric study that gives insight into the influence of these parameters on the bulging factors is of great value. In this paper, we present selected results of such a study and summarize a reliable methodology to obtain the bulging factors.

Keywords: Bulging; Crack propagation; Pressurized shell; Locking; Stress intensity factors; Shell modeling

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1. Introduction

The propagation of cracks in pressurized fuselages is a subject of great interest for the aero-astro industry. Both the stable crack growth due to fatigue and the unstable crack growth causing catastrophic failure can be studied by means of crack tip stress intensity factors. Focusing on the stress intensity factor K_I, it is well known that a plate subjected to in-plane tension displays a smaller K_I than the K_I obtained for a cylindrical shell subjected to internal uniform pressure, such that the stress states far from the crack are analogous, as shown schematically in Fig. 1.

Therefore, the ratio

$$\beta = \frac{K_I^{shell}}{K_I^{plate}} \tag{1}$$

is greater than 1. The reason for this is that in the crack region, a shell model solution reflects a coupled membrane-bending behavior, leading to larger crack opening displacements and, hence, larger K_I than that of the plate. The shell transverse displacements are known as bulging displacements and β is referred to as the bulging factor. Typical bulging displacements are shown in Fig. 2.

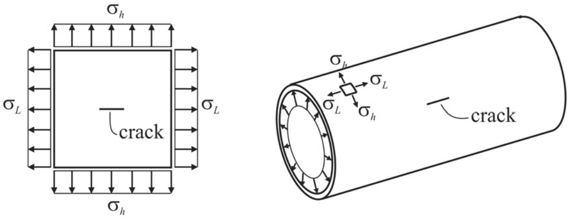

Fig. 1. Flat and curved stress conditions.

Bulging factors for specific situations were derived analytically, many years ago, based on linear shell theory. It turns out that the linear shell theory overestimates the bulging displacements and, consequently, the bulging factors. In fact, there is a stiffening effect due to a geometric nonlinear behavior, which is significant at the crack region. To obtain the nonlinear bulging factors, a finite element approach is, in general, adopted. We note, however, that for the situations of practical interest, i.e. associated with real fuselages, the shell is very thin. It is usual to have values for the shell thickness (t) of the order of 1 mm and radius (R) of the order of 1500 mm ($t/R = 1/1500$). Therefore, it is imperative to use shell finite elements that are totally

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[*] Corresponding author. Tel.: +55 11 3091 5205; Fax: +55 11 3091 5181; E-mail: mlbucale@usp.br

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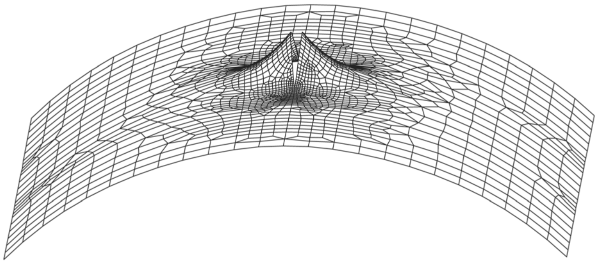

Fig. 2. Typical bulging displacements (magnification factor ≡ 10).

free from locking. The methodology presented uses shell elements based on the mixed interpolation of the tensorial components approach, namely the MITC4 [1] and MITC9 [2] elements, which have been studied thoroughly, being free from locking. These elements are available in ADINA [3].

Considering a real fuselage, the bulging factors depend on a number of parameters, such as pressure level, skin thickness, structural area of the stiffeners, and distance between stiffeners, among others. A parametric study that gives insight into the influence of these parameters on the bulging factors as well as numerical predictions is of great value. Such a parametric study has been performed, and the objective of this paper is to present selected results of this study and to summarize a reliable methodology to obtain the bulging factors.

2. Methodology to evaluate K_I

The approach we use to obtain the K_I is the modified crack closure integral method. This is based on the energy release rate G_1 (for mode *I*), which is related to the stress intensity factor by

$$G_1 = \frac{(K_I)^2}{E} \tag{2}$$

where E is the Young's modulus.

The energy release rate G_1 is evaluated from the energy necessary to close the crack, assuming that the crack tip geometrical profile is maintained in a crack increase. In finite element analysis, this work is evaluated using nodal forces and displacements [4].

We remark that the same procedure can be used for both linear and nonlinear analysis. In the nonlinear analysis, an incremental nonlinear analysis is performed up to the desired load level, and then the procedure is applied.

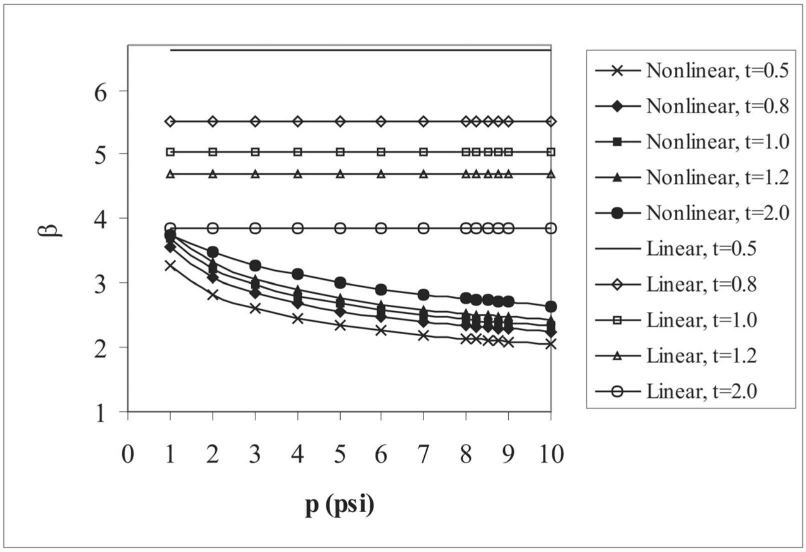

Fig. 3. Bulging factors for pressurized unstiffened cylindrical shell for several values of thickness *t*.

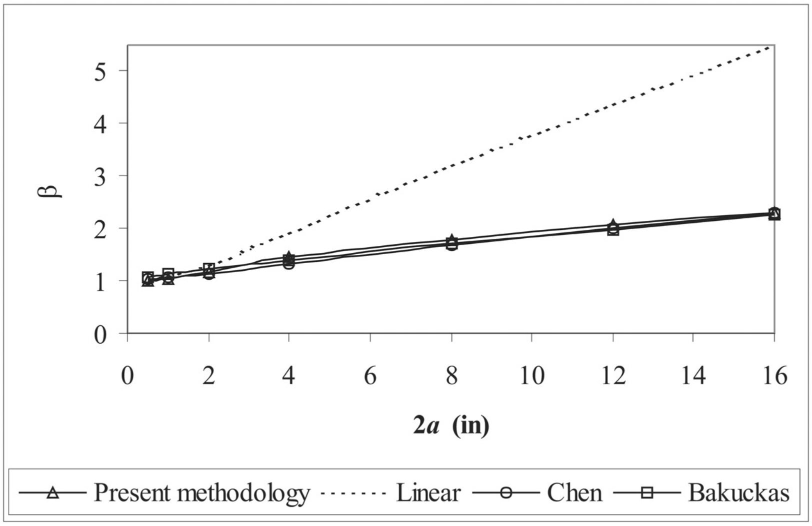

Fig. 4. Comparison of bulging factors.

3. Numerical results

In this section, we report selected results from the bulging factor parametric study. The factors K_I^{shell} and K_I^{plate} are obtained by finite element analysis using the MITC4 element.

3.1. Unstiffened cylindrical shell

We consider a cylindrical shell of radius $R = 1.505$ m subjected to an internal pressure in the range $1 \leq p \leq 10$ psi, with a longitudinal crack of length $2a = 16$ in.

We are interested in the influence of the shell thickness on the bulging factors. In Fig. 3, we show the bulging factors for different levels of pressure and different values of thickness. Let us focus first on the thickness $t = 0.5$ mm. Of course, for the linear model, the bulging factor does not depend on the pressure. The well-known reason is that in a linear model, both the stress level and the bulging displacements are proportional to the pressure, always leading to the same value of β. The bulging factor for the nonlinear model clearly depends on the pressure and β decreases with increasing pressure, since the stiffening effect also increases with pressure. We note that for this thickness, there is a huge difference in the bulging factors (linear and nonlinear), even for the smallest pressure of $p = 1$ psi, i.e. $\beta = 6.62$ for the linear model while $\beta = 3.26$ for the nonlinear case.

Considering now the change in thickness, the linear bulging factor decreases with increasing thickness, since the bulging displacements become smaller as the thickness grows. However, for the nonlinear model, we recognize two effects. On the one hand, as the thickness decreases, the bulging displacements tend to increase. On the other hand, for the smaller thickness, the nonlinear stiffening effect due to pressure is greater. Since these two effects have opposing influences on the value of the bulging factor, we can not, a priori, anticipate which is the actual tendency. The results of Fig. 3 reveal that the stiffening effect due to the pressure prevails, since for a fixed value of the pressure, β decreases with a decrease in thickness.

To assess our results, we compare them with those of Bakuckas et al. [5] and Chen [6] for the thickness $t = 0.8$ mm and for pressure 8.5 psi. We obtain close predictions, as shown in Fig. 4.

3.2. Stiffened cylindrical shell

We consider now the shell described above with transverse stiffeners spaced by 425 mm. Each stiffener has a cross-sectional area of 84.3 mm^2 and a moment of inertia of 2.777×10^4 mm^4. The center of gravity is 56.65 mm from the shell midsurface. The stiffeners are modeled as beam elements, and rigid links are used to connect them to the shell.

In Fig. 5, we show results that are analogous to those of Fig. 3. Note that the bulging factors are significantly lower than those for the unstiffened shell. For the linear models, the same trend as observed previously is displayed, i.e. β increases with decreasing thickness. We note, however, that this trend is also observed for the nonlinear analysis. In this case, due to the greater initial stiffness of the shell, the nonlinear stiffening effect due to the pressure is not enough to compensate for the

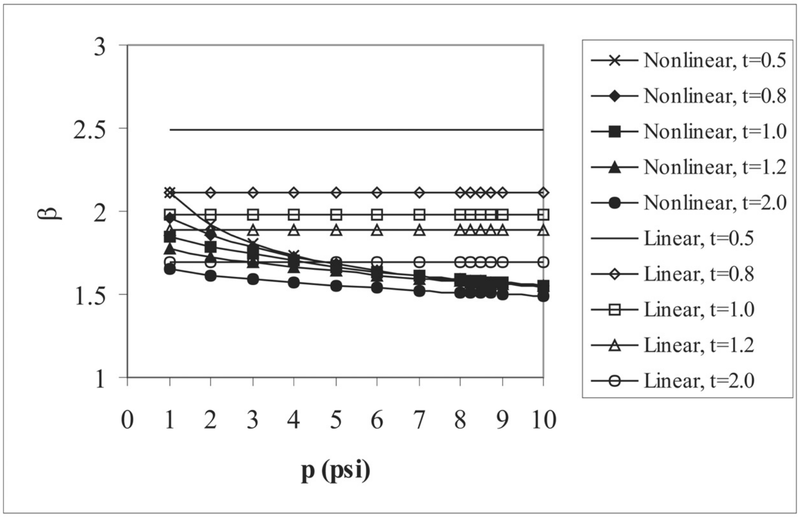

Fig. 5. Bulging factors for pressurized stiffened cylindrical shell for several values of thickness t.

increase of bulging displacement associated with the thickness decrease alone. We note that the difference in bulging factor predictions decreases with increasing pressure.

4. Conclusions

We have summarized a methodology to obtain bulging factors for unstiffened and stiffened pressurized cylindrical shells.

Locking free shell elements are a requirement for reliable results, since the problem is very much prone to locking. The MITC4 shell element, which does comply with the locking free requirement, was used.

Selected results of a comprehensive parametric study are shown, which point out the interesting behavior of the bulging factors when various shell thickness are considered.

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