Application of joint time–frequency representation method in transient analysis of semi-infinite media

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Abstract

Many methods have been developed for the analysis of semi-infinite media in highly idealized situations where either nonlinear behavior of the near field was neglected or a full nonlinear analysis was conducted with heavy computational efforts. In this work, to model a half-space medium with various irregularities, a combination of finite and infinite elements has been used. More problems arise when nonlinearities are going to be included in the system. The impedance matrix of an infinite element is frequency dependent [1,2] while for nonlinear analyses the equations of motion should be solved in time domain [3]. To overcome this difficulty the time-frequency representation of signals is utilized [4,5] to investigate the behavior and frequency content of infinite elements' responses and obtain the dominant frequencies of their ends' vibrations at each time interval. Using the weighted average approach, the effective stiffness, mass and damping matrices of infinite elements are then assembled into the matrices of the system at each time interval. The matrices of infinite elements will be piecewise constant, and hence they can be transferred easily into time domain.

Keywords: Infinite elements; Time–frequency representation; nonlinear analysis; Adaptive optimal kernel; Short time ambiguity function

1. Introduction

Some of the most widely used methods in modeling semi-infinite media are boundary elements, infinite elements, thin layer elements and simple constant stiffness–damping methods. In some of these methods the impedance matrix of the far field is approximated and then transformed into time domain to develop nonlinear analysis. The stiffness, mass and damping matrices (including material damping and radiation damping) of the far field are included in the impedance matrix. In this research an infinite element method, which was developed by Zhao and Valliappan [6,2] is used. If the frequency content of the response of these infinite elements in a given time interval is determined, then the effective stiffness, mass and damping matrices of infinite elements can be evaluated. In fact, due to the uncertainty principle of signal analysis, if the resolution of spectrum in time domain increases, the resolution in frequency domain decreases and vice versa. A better way is to obtain response spectrum in a time interval and then make a judgment about dominant frequencies of the response. It should be noted that, in this procedure, time interval is different from time step of nonlinear analysis. The response spectrum of each infinite element changes with time. For the time–frequency representation (TFR) part, adaptive optimal-kernel time–frequency representation of signals is used [7,8].

2. TFR based on adaptive optimal kernel

Time–frequency representations with fixed windows or kernels are used in many applications, but perform well only for limited classes of signals. However, representations with signal-dependent kernels can overcome this limitation [9]. This method is very flexible to detect localized behaviors of signals and on-line implementation.

To measure the frequency spectrum in a time interval, there is no need to analyze the responses from the beginning of excitation up to the current time. Simply, a small tail of information from the response spectrum of each infinite element should be processed. It significantly reduces the computational cost in the TFR part. In each time interval, the frequencies, which have more energy in

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the response spectrum, have more significant effects on stiffness, mass and damping matrices of infinite elements.

The method proposed by Jones and Baraniuk [7,8], includes a signal-dependent TFR based on a radially Gaussian kernel, which computes the ambiguity function (AF) of the entire signal and then determines the optimal kernel based on that AF. The ambiguity function includes the frequency information of the entire signal; therefore, in order to discover the local characteristic of the signal, a proper window on the AF is needed. This running-time algorithm is called short time ambiguity function and is very helpful to detect the characteristics of the signals that change over time.

Around the most recent time at which analysis has been done, a short segment of response is selected and its time-frequency spectrum is calculated. Therefore, the frequency content of the response in that segment can be obtained and the effective matrices will be produced by this spectrum.

3. Frequency dependent infinite elements

To simulate non-reflecting boundaries for semi-infinite media, the dynamic infinite element, developed by Zhao and Valliappan, has been used [2]. Using this type of infinite element, transmitting boundaries for P, SV, SH and Rayleigh waves can be modeled. This is a 6-node infinite element with a harmonic decaying exponential shape function included inside its kernel to simulate the decaying behavior in infinite direction (Fig. 1).

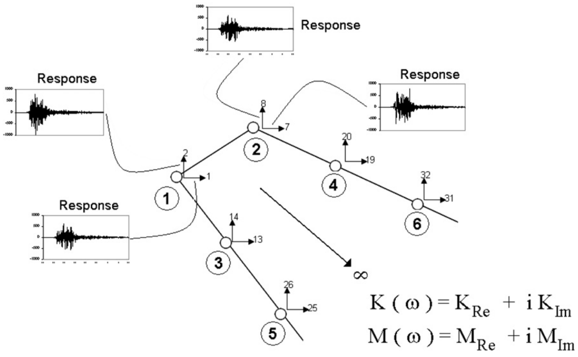

Fig. 1. Frequency dependent infinite element.

$$\Psi_i = P_i(\xi)\frac{1}{2}(1-\eta)(i = 1,3,5)$$
$$\Psi_i = P_i(\xi)\frac{1}{2}(1+\eta)(i = 2,4,6) \quad (1)$$

In which:

$$P(\xi) = e^{-\alpha\xi}\left(C_1 e^{-i\beta_1\xi} + C_2 e^{-i\beta_2\xi} + C_3 e^{-i\beta_3\xi}\right)$$

β_i = wave numbers for R − wave, S − wave and P − wave

C_i = constants (2)

Constants C_i should be calculated from the method proposed in [2].

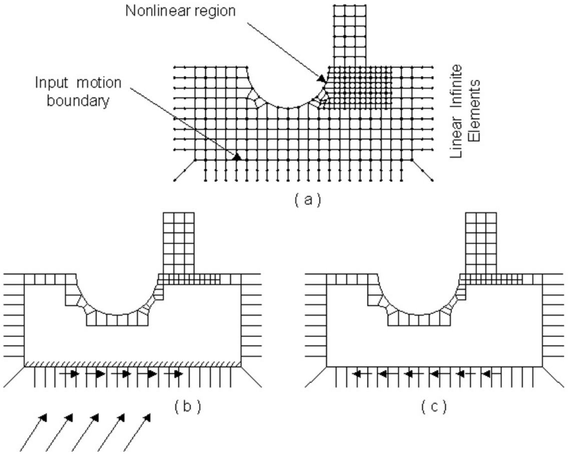

Fig. 2. Modeling of semi-infinite media.

4. Wave input modeling in time domain for P and SV-waves

To simulate wave scattering problems, the method presented in [6] has been used. It is based on elastic wave motion theory and superposition concept. In order to obtain the response of the near field due to the incident wave from the far field, the model shown in Fig. 2(a) can be divided into two parts as shown in Figs. 2(b) and 2(c). In Fig. 2(b), an artificial fixed boundary is added onto the wave input boundary, so that the incident wave reflects, and reaction forces Γ will appear on this fixed boundary. Adding the opposite of Γ on the input boundary in Fig. 2(c), the effect of the fixed boundary can be eliminated. Generalized stresses vector due to SV and P-wave incidence on the wave input boundary (Fig. 2) is represented as [6]:

$$\{\sigma\}^e = \begin{Bmatrix} \sigma_y^{SV} \\ \sigma_{xy}^{SV} \end{Bmatrix} = -\begin{Bmatrix} f_\sigma^{SV} \\ f_\tau^{SV} \end{Bmatrix} \frac{E\omega}{C_{SV}} i\exp\left(-i\frac{\omega}{C_{SV_x}}x\right) \quad (3)$$

$$\{\sigma\}^e = \begin{Bmatrix} \sigma_y^{P} \\ \sigma_{xy}^{P} \end{Bmatrix} = -\begin{Bmatrix} f_\sigma^{P} \\ f_\tau^{P} \end{Bmatrix} \frac{E\omega}{C_{P}} i\exp\left(-i\frac{\omega}{C_{P_x}}x\right) \quad (4)$$

In which f_σ^{SV}, f_σ^{P}, f_τ^{SV} and f_τ^{P} are the stress factors [6].
E = Elasticity modules
v = Poison's ratio
C_{SV} = SV-wave velocity
C_p = P-wave velocity
ω = Circular frequency
x = x coordinates of input nodes

The stresses and effective load vector on the wave input boundary, which are produced by aforementioned equations are frequency dependent, but transformable into time domain by using inverse fast Fourier transformation (IFFT).

5. Mean instantaneous stiffness, mass and damping

To transform matrices into time domain the weighted average method is used. As it can be observed from Fig. 3 and 4, in a signal like an earthquake, at a specific time inside the power spectrum diagram, just a few frequencies are dominant. The effective stiffness matrix can be calculated for this range of frequencies (Fig. 5). The same method should be used to assemble effective mass and damping matrices.

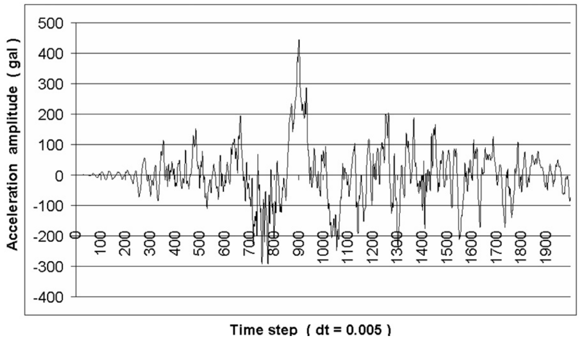

Fig. 3. Example of an earthquake excitation.

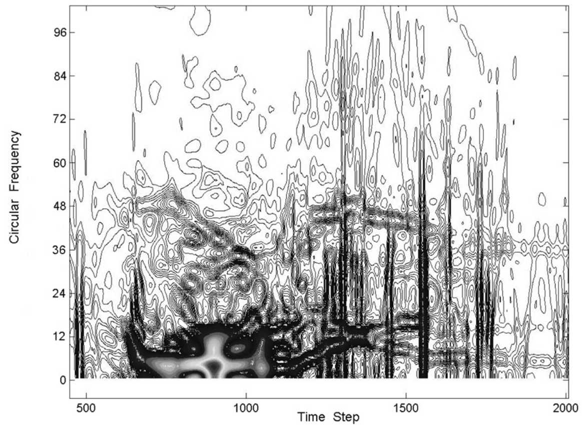

Fig. 4. Time dependent spectrum of excitation.

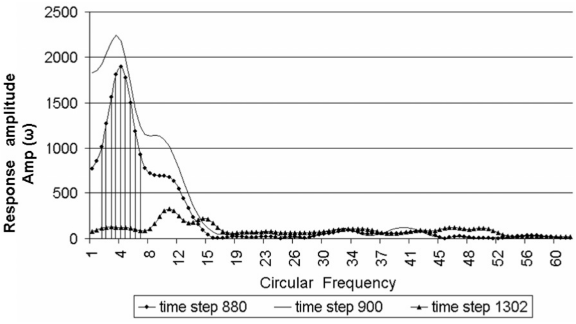

Fig. 5. Response spectrum of an infinite element in different times.

Equation of motion of the system in frequency domain is written as:

$$\left(-\omega^2[\mathrm{M}(\omega)] + \left(1 + \hat{\mathrm{i}}\eta_\mathrm{d}\right)[\mathrm{K}(\omega)]\right)\{\tilde{\mathrm{u}}(\omega)\} = \{\mathrm{F}(\omega)\} \qquad (5)$$

Where:

$$[\mathrm{K}(\omega)] = [\mathrm{K}_{\mathrm{Re}}(\omega)] + \hat{\mathrm{i}}[\mathrm{K}_{\mathrm{Im}}(\omega)] \qquad (6)$$

$$[\mathrm{M}(\omega)] = [\mathrm{M}_{\mathrm{Re}}(\omega)] + \hat{\mathrm{i}}[\mathrm{M}_{\mathrm{Im}}(\omega)] \qquad (7)$$

$$[\mathrm{Z}(\omega)] = -\omega[\mathrm{M}_{\mathrm{Im}}(\omega)] + \frac{1}{\omega}[\mathrm{K}_{\mathrm{Im}}(\omega)] \qquad (8)$$

η_d = Hysteretic damping coefficient of the medium
$[\mathrm{Z}(\omega)]$ = Intermediate matrix for transforming damping into time domain
$\{\mathrm{F}(\omega)\}$ = Load vector in frequency domain

Using weighted average scheme on matrices:

$$\text{Instantaneous stiffness}[\mathrm{K}]^{\Delta t} = \frac{\Sigma\left(\mathrm{Amp}^{\Delta t}(\omega_i)[\mathrm{K}_{\mathrm{Re}}(\omega_i)]\right)}{\Sigma\left(\mathrm{Amp}^{\Delta t}(\omega_i)\right)} \qquad (9)$$

$$\text{Instantaneous mass}[\mathrm{M}]^{\Delta t} = \frac{\Sigma\left(\mathrm{Amp}^{\Delta t}(\omega_i)[\mathrm{M}_{\mathrm{Re}}(\omega_i)]\right)}{\Sigma\left(\mathrm{Amp}^{\Delta t}(\omega_i)\right)} \qquad (10)$$

$$\text{Instantaneous damping}[\mathrm{C}]^{\Delta t} = \frac{\Sigma\left(\mathrm{Amp}^{\Delta t}(\omega_i)[\mathrm{Z}(\omega_i)]\right)}{\Sigma\left(\mathrm{Amp}^{\Delta t}(\omega_i)\right)} \qquad (11)$$

To reduce the error estimation of frequency content, it is better to evaluate response spectrum in the last time interval Δt (Fig. 6). Length of tail shows the largest wavelength of signal's components that should be detected during vibration. The results of this method are constant matrices in a sliced interval of time-dependent spectrum. These piecewise constant matrices are called mean instantaneous stiffness, mass and damping (Fig. 7), and should be added to matrices of finite element

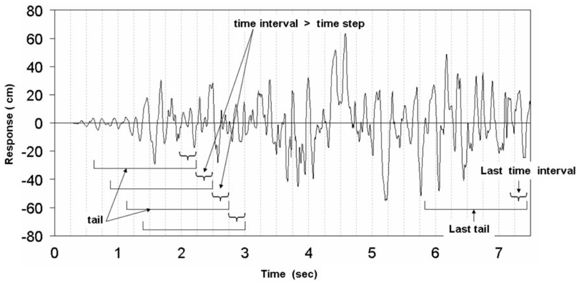

Fig. 6. On-line tracking of response in last time interval.

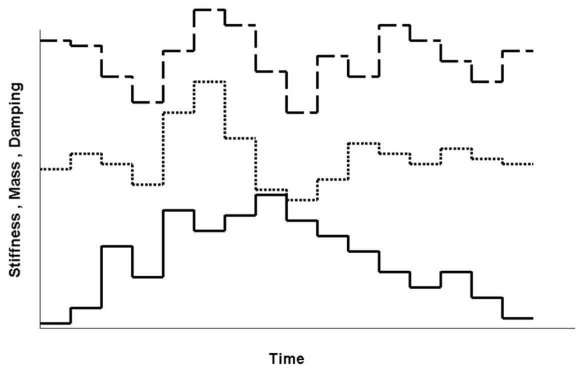

Fig. 7. Piecewise constant stiffness, mass and damping matrices.

part. If the response spectrum of the far field is close enough to the response spectrum by which instantaneous matrices were calculated, then the tail is shifted to a new time otherwise the mean instantaneous matrices should be updated by the new spectrum and the procedure should be repeated. These steps are repeated until the end of vibration. Nonlinearities in the near field are included in the FEM part of the system.

6. Conclusion

The proposed method to model semi-infinite media in time domain, is a combination of finite–infinite elements and adaptive optimal kernel time–frequency representation method. The merit of this method is that the accuracy of infinite elements can be linked to the simplicity of the method, which considers constant stiffness and dashpot. In previous methods it is common to consider real and imaginary parts of the impedance matrix as an equivalent stiffness and damping matrices in time domain respectively. This mixture is correct as far as steady state waves are considered. But for transient waves it will produce inaccuracy, because the mass has not been considered for the far field. The proposed method introduces stiffness, mass and damping matrices separately in time domain.

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