On the forced dynamics of floating plates

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Abstract

The uplift of an infinite plate floating on a fluid of finite depth is studied with a view to inverting the forcing required to produce prescribed dynamics. The problem is ill posed and the potential of the forcing is the solution of a Volterra integral equation of the first kind and of convolution type. Analysis shows that the degree of ill-posedness is moderate and reveals that the initial rate of forcing is proportional to the initial acceleration of the plate. For error-free data, this analytical result, and a nonuniform mesh, are used to numerically compute the forcing to three decimal places.

Keywords: Volterra integral equation of the first kind; Ill-posed; Canonical problems

1. Introduction

A schematic of a floating plate is shown in Fig. 1. This figure shows an infinite plate of thickness h, floating on a fluid of finite depth $H\ell \gg h$, and subject to the uplift pressure $\rho_f ghP(\tau)\delta(r)/r$. Here H is the nondimensional depth, ρ_f is the fluid density, g is the acceleration due to gravity, and P is the nondimensional force at nondimensional time τ. The radial coordinate r and vertical coordinate z are dimensionless. The characteristic length ℓ is defined in terms of the flexural rigidity D of the plate by

$$\ell = \left(\frac{D}{\rho_f g}\right)^{1/4} \tag{1}$$

Physical time is $t = \tau\sqrt{\ell/g}$. The uplift displacement at $r = 0$ and at time τ is $hw(\tau)$. Again w is nondimensional. The parameters of the problem are the nondimensional depth H and the dimensionless plate-to-fluid mass ratio

$$\mu = \frac{\rho_p h}{\rho_f \ell} \tag{2}$$

The plate is assumed to be thin and linearly elastic and the fluid is assumed to be incompressible, nonviscous and irrotational. Since large displacements are outside the realm of these assumptions, only a finite time interval is physically meaningful, say $0 \le \tau \le a$.

At time zero, the midsurface of the plate lies in the $z = 0$ plane, the plate is stationary, $w(0) = 0$ and $w'(0) = 0$, and the forcing is zero, $P(0) = 0$. During uplift, the force potential $\phi(\tau) = P'(\tau)$ is the solution of the Volterra integral equation

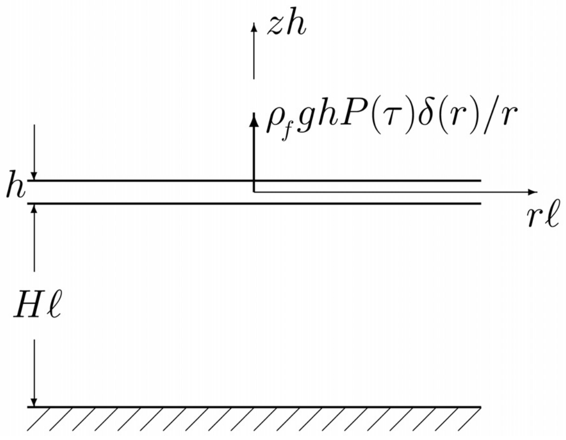

Fig. 1. Floating plate schematic.

$$\int_0^\tau k(\tau - u)\phi(u)\, du = w(\tau), \quad 0 \le \tau \le a \tag{3}$$

where

$$k(\tau) = \frac{\pi}{4} - \int_0^\infty \frac{x}{1 + x^4}\cos[\beta(x)\tau]\, dx$$

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$$\beta(x) = \sqrt{\frac{1+x^4}{\mu + \frac{1}{x\tanh(Hx)}}} \tag{4}$$

In Dempsey et al. [1], the time interval [0, a] (for $a = 10$) is discretized into n equal intervals (for successively higher values of n) and trapezoidal integration is used to approximate Eq. (3) by a succession of linear systems

$$\boldsymbol{A}\phi = \boldsymbol{w} \tag{5}$$

It is determined that growth of the 2-norm condition number of $\boldsymbol{A}$ is $O(n^{1.87})$ which is concomitant with a moderate degree of ill-posedness.

It is also shown in Dempsey et al. [1] that, for a prescribed uplift displacement function $w(\tau)$, the initial rate of forcing is related to the initial acceleration of the plate by

$$P'(0) = \frac{4\sqrt{\mu}}{\pi} w''(0) \tag{6}$$

In Vasileva and Dempsey [2], this exact result for $\phi(0)$ is incorporated into a numerical scheme whereby a non-uniform mesh and trapezoidal integration is used to demonstrate convergence to three decimal places of accuracy for the uplift forcing $P(\tau)$, which is computed from the potential $\phi(\tau)$ by integration. The numerical results in [2] are computed for two canonical problems. Also, in [1,2], a model problem is used extensively as an analytical and computational benchmark. Descriptions of the canonical problems and the model problem follow.

2. Canonical problems

Most of the practical uplift scenarios are covered by studying two canonical problems that were outlined for the first time in Dempsey and Vasileva [3]. In one, the upward velocity is virtually constant, while, in the other, the upward acceleration is constant.

2.1. Constant velocity (CV)

In the first canonical problem, the center of the plate is forced upwards at a ‘constant’ velocity so that

$$w(\tau) = V\left(\tau - \frac{1-e^{-\alpha\tau}}{\alpha}\right), \quad w'(\tau) = V(1-e^{-\alpha\tau}),$$
$$w''(\tau) = \alpha V e^{-\alpha\tau} \tag{7}$$

The exponential term is required in order to ensure that $w'(0) = 0$, otherwise $w'(\tau) \approx V$. In this paper, $\alpha = 50$, a value based on experimental data.

2.2. Constant acceleration (CA)

In the second canonical problem, the center of the plate is forced upwards at a constant acceleration so that

$$w(\tau) = \frac{1}{2}A\tau^2, \quad w'(\tau) = A\tau, \quad w''(\tau) = A \tag{8}$$

3. Model problem

The model problem was introduced in [3], for $\mu = 1$. The logic behind it is that $\beta(x) \sim x^2/\sqrt{\mu}$ as $x \to +\infty$. If this asymptotic result is substituted for β in Eq. (4), the model problem

$$\int_0^\infty \kappa(\tau-u)\phi(u)\,du = w(\tau), \quad 0 \le \tau \le a \tag{9}$$

with

$$\kappa(\tau) = \frac{\pi}{4} - \int_0^\infty \frac{x}{1+x^4}\cos\left(\frac{x^2\tau}{\sqrt{\mu}}\right)dx = \frac{\pi}{4}\left(1 - e^{-|\tau|/\sqrt{\mu}}\right) \tag{10}$$

is obtained. The latter has the advantage of being analytically tractable and captures much of the physics of the parent plate problem. The solution of the model problem comprised by Eqs. (9) and (10) is given by

$$\phi(\tau) = \frac{4}{\pi}\left(w' + \sqrt{\mu}w''\right), \quad P(\tau) = \frac{4}{\pi}\left(w + \sqrt{\mu}w'\right) \tag{11}$$

Applying Eq. (11) to the constant velocity canonical problem in Section 2 gives the model problem solutions for ϕ and P as

$$\phi_{\mathrm{CV}}(\tau) = V\frac{4}{\pi}\left[1 + \left(\alpha\sqrt{\mu} - 1\right)e^{-\alpha\tau}\right] \tag{12}$$

$$P_{\mathrm{CV}}(\tau) = V\frac{4}{\pi}\left[\tau + \left(\sqrt{\mu} - \frac{1}{\alpha}\right)(1 - e^{-\alpha\tau})\right] \tag{13}$$

while for the constant acceleration canonical problem, the solutions are

$$\phi_{\mathrm{CA}}(\tau) = A\frac{4}{\pi}\left(\sqrt{\mu} + \tau\right), \quad P_{\mathrm{CA}}(\tau) = A\frac{4}{\pi}\left(\frac{\tau^2}{2} + \sqrt{\mu}\tau\right) \tag{14}$$

These results, for the model problem, predict the shape of the curves for ϕ and P for the actual plate problem.

4. Numerical details

In [2], it was found that numerical computations for both canonical problems benefit from a nonuniform

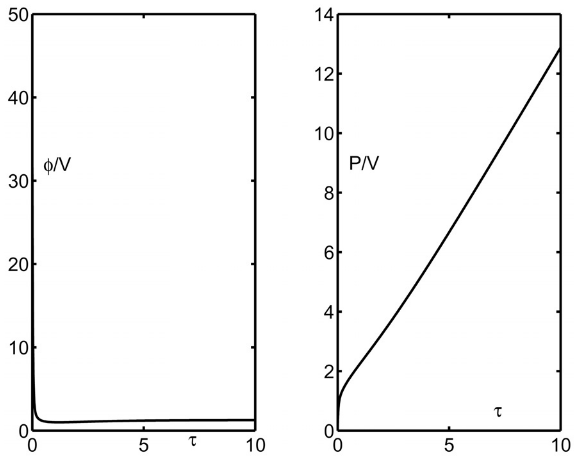

Fig. 2. Curves of ϕ/V and P/V for the constant velocity plate problem (for $\mu = 0.15$ and $H = 1$).

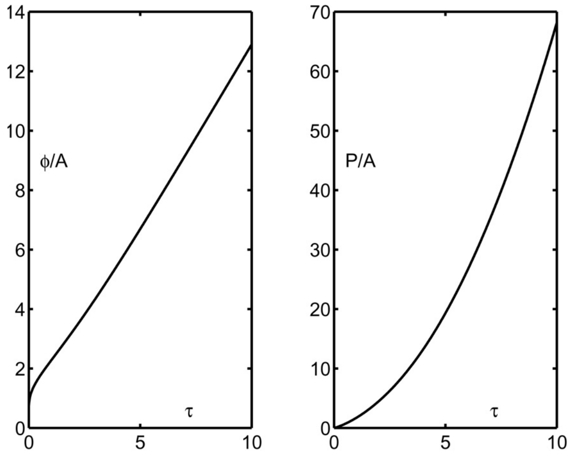

Fig. 3. Curves of ϕ/A and P/A for the constant acceleration plate problem (for $\mu = 0.15$ and $H = 1$).

mesh that has nodes clustered near the origin. Equation (13) for the model problem was used in [2] to generate this mesh for use with the plate problem. Curves for the plate problem for the constant velocity and constant acceleration canonical problems are shown in Fig. 2 and 3, respectively. The numerical details are discussed at length in [2].

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