Transient dynamic analysis over the low- and medium-frequency ranges for engineering structures

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Abstract

The paper deals with a new computational method for transient dynamic analysis which enables one to cover both the low- and medium-frequency ranges. This is a frequency approach in which the low-frequency part is obtained through a classical technique, while the medium-frequency part is handled through the variational theory of complex ray (VTCR) initially introduced for vibrations. The aim of the paper is to extend the capabilities of the method to engineering systems.

Keywords: Dynamics; Frequency domain; Medium frequencies; Computation; VTCR; Multiscale

1. Introduction

The response of industrial structures, especially during the transient stage, cannot be completely described using current tools: the medium-frequency range is often ignored unless the calculation is carried out with very refined spatial and time meshes as mentioned in Bathe [1] or in Belytschko et al. [2]. But taking the medium-frequency content into account can prove necessary since, although the displacements are small over this frequency range, the kinetic energy can be significant. Transient dynamic analysis in this frequency range for complex engineering structures presents an important challenge. The present work, using new computational strategies in dynamics, answers this challenge for the transient part of the solution. The problem is solved in the frequency domain. One needs to solve a forced vibration problem over a frequency range which includes the low- and medium-frequency ranges.

The main problem resides in the resolution of the forced vibration problem over the medium-frequency range. The alternative approach we use here, called the 'variational theory of complex rays' (VTCR), was introduced in Ladevèze [3] for the calculation of medium-frequency vibrations.

Until now the present frequency approach has been applied on academic examples made of assemblies of beams [4,5] and shows the importance of the medium frequencies. The central point of this paper is to extend the capabilities of the method to problems that tend towards complex engineering structures, assemblies of plates, shells, and beams submitted to impacts or moving loadings, while decreasing the cpu time.

2. Frequency analysis of the reference problem

The transient dynamic problem is written as a global variational problem over the frequency–space domain. Thus, the Fourier transform is applied to all time-dependent quantities, yielding frequency-dependent functions. One then needs to solve a forced vibration problem over a wide frequency range $[0, \omega'_c]$.

The present approach considers a partition of the frequency range $[0, \omega'_c]$ being studied into two parts: a low-frequency part $[0, \omega_c]$ and a medium-frequency part $[\omega_c, \omega'_c]$. Outside of this frequency range, the kinetic and strain energies in the structure are assumed to be negligible. For the low-frequency range $[0, \omega_c]$, the frequency response function is obtained using a finite element technique: it is advantageous to use a reduced basis constructed from the first vibration modes and completed with the static modes. The difficulty for the modeling and calculation of medium-frequency vibrations lies in the fact that the wavelengths of the phenomena being studied are very small compared with the characteristic dimensions of the structure. The

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VTCR is a suitable computational method for the medium-frequency range $[\omega_c, \omega'_c]$.

The final solution in the space – time domain is given using the inverse Fourier transform.

3. Basics of the VTCR

The variational theory of complex rays is an approach dedicated to the calculation of medium-frequency vibrations. It can be summarized in two major points:

- The first characteristic of this approach is the use of a new variational formulation of the problem being considered, which enables one to use *a priori* independent approximations within each substructure. In other words, the transmission conditions on the displacements as well as the stresses at the interfaces between substructures do not need to be verified *a priori*, but are built into the variational formulation.
- The second characteristic of the VTCR is the introduction within each substructure of two-scale approximations with a strong mechanical meaning: the solution is assumed to be well-described locally in the neighborhood of a point X as the superposition of an infinite number of local vibration modes. These basic modes (which can be interior modes, edge modes or corner modes) verify the governing equation and the constitutive law over the substructure's domain. All wave directions are taken into account and the unknowns are discretised amplitudes with relatively long wavelengths.

The feasibility and effectiveness of the method are demonstrated in Ladevèze et al.[6] and in Rouch et al.[7].

In our proposed approach to the transient dynamic response analysis, it is advantageous to use the VTCR over a relatively wide frequency range. The idea is to introduce a two-scale approximation in terms of the frequency and then to partition the coefficients of the linear system into 'slowly-' and 'fast-varying' coefficients with respect to the frequency. Then an ad hoc strategy uses the mean value and the complementary part of the solution over the frequency band to predict the behavior of the quantities of interest.

4. Effectiveness of the method

Figure 1 shows an academic example consisting of a slightly damped plate subjected to an impact bending. The velocity in the middle of the plate obtained with the presented frequency-domain approach, with and without the medium-frequency range, is compared with the velocity obtained with a standard time-domain method in Fig. 2. This example shows that the presented approach yields good results provided that the medium-frequency content is taken into account. Therefore, although the displacements due to the medium-frequencies are small, one cannot leave out the medium-frequencies, as these appear to a great extent in the velocity and, consequently, in the kinetic energy.

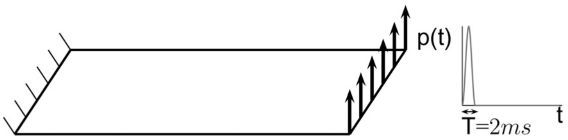

Fig. 1. A plate subjected to an impact bending.

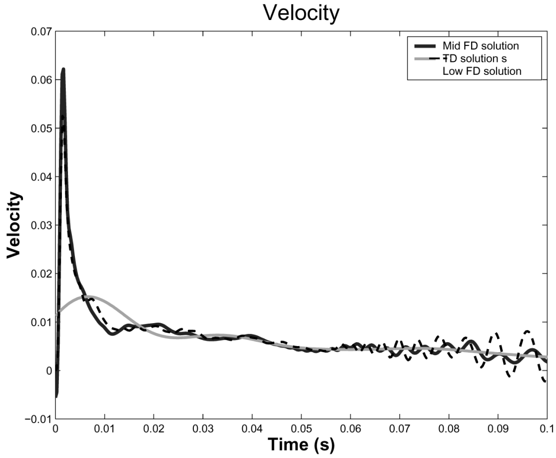

Fig. 2. Velocity in the middle of the plate.

In the approach, we propose to consider the time response as the superposition of two components that stem respectively from the low- and the mid-frequency contributions. These two components can then be computed separately: the low-frequency contribution can easily be obtained using any adapted technique and the medium-frequency contribution, obtained using the present approach, can be added afterwards. This enables us to use the fast Fourier transform, see Brigham [8], on the medium-frequency range only and to take advantage of the quick fading-out of these medium-frequencies in scattering media, and thus to dramatically reduce the computation cost.

Table 1 summarises the size of the computation problems for the both methods: the frequency-domain approach proposed in this paper, and the classical time-domain approach. The present approach is now being applied on assemblies of plates submitted to moving impact loadings. The current work aims at studying the possibilities of the method for complex industrial structures, such as the launchers submitted to pyrotechnical shocks for instance.

Table 1
Size of the computation problems for the time-domain (TD) and the frequency-domain (FD) approaches

Approach	TD explicit scheme	FD Reduced basis	VTCR
DOFs	10000	112	32
Number of calculations	200000	2800	1000

5. Conclusions

A new theory for the calculation of transient dynamic responses with a high frequency content has been developed and its effectiveness has been shown on problems that tend towards being engineering problems. The point of this technique is to restitute both the low- and the medium-frequencies while decreasing the cpu time on complex engineering structures. In this paper, non-linearities in the frequency domain only have been considered. The extension to non-linearities expressed in the time domain is the subject of further developments.

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