

# Computational and experimental study of active vibration control of a rectangular plate coupled to liquid

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## Abstract

Active structural vibration control of a rectangular plate representing a flexible wall of a thick Plexiglas container is investigated, for both empty and water-filled tank. A finite-elements model of the plate, which includes additional masses and stiffness of actuators and sensors, is presented. The computational results are compared with experimental modal analyses. A filtered-x LMS adaptive feedforward algorithm is experimentally applied with a SISO approach on the perturbed system, obtaining satisfactory results on the control of the first three modes.

*Keywords:* Active control; Vibrations; Finite elements; Fluid–structure interaction; Feedforward

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## 1. Introduction

The suppression of structural vibrations of very light structures is a topic of great interest in many applications, especially in aeronautical and space structures, which often also present thin walls containing fuel. The presence of the liquid increases the modal mass of the system and free surface waves complicate the system dynamics, as reported by Amabili [1] and Morand and Ohayon [2].

In the present study, the active vibration control of a thin-walled rectangular plate representing a flexible wall of a Plexiglas container is investigated in the case of both empty and water-filled tank. The secondary control input is generated by piezoelectric PZT actuators, as suggested by several investigators [3,4]. Sensors and piezoelectric patches are positioned in a nearly-collocated configuration [5,6].

The dynamics of the system is investigated by carrying out experimental modal analyses for different levels of fluid and comparing such results with those given by a 3D finite-elements model. A filtered-x LMS adaptive feedforward algorithm, as described by Fuller et al [7], is finally applied to the system in a SISO (Single Input

Single Output) approach with both empty tank and water-filled tank.

## 2. Experimental modal analyses

The system consists of a rectangular aluminium plate ( $400 \times 283 \times 0.8$  mm), which is connected by bolts to the thick wall of a Plexiglass rectangular container (Fig. 1), in correspondence of an opening of dimensions  $350 \times$

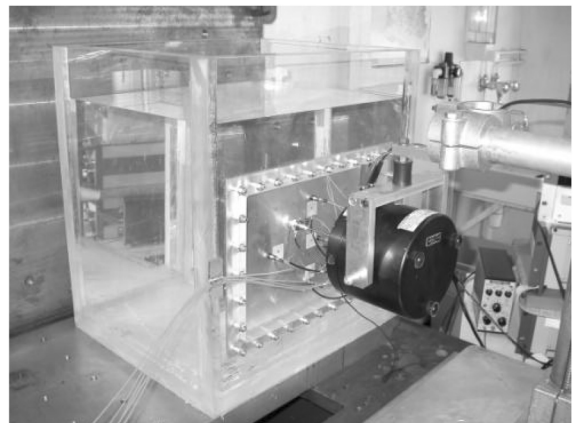


Fig. 1. Experimental set-up (water-filled tank).

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233 mm. The plate has boundary conditions sufficiently close to those of clamped edges.

An electro-dynamical shaker excites the plate in eccentric position and the force signal is registered by a force transducer glued to the plate. The system response is measured on a grid of  $9 \times 7$  points by using a sub-miniature accelerometer. Modal analyses are performed on the plate before and after applying the control components, in the cases of empty tank, tank filled to half-plate, and tank full of water. As the water level is increased, a progressive reduction of the values of the natural frequencies is observed. Mode shapes change, but less significantly. Introducing PZT patches and accelerometers produces the same sequence of modes.

### 3. Finite-elements model

A 3D finite-elements model of the plate has been built in order to predict the dynamic behaviour of the system and to compare such results with the experimental modal analyses. The effects of fluid-structure interaction are at present not included in the model. Clamped boundary conditions are assumed.

The load transducer, the PZT patches and the accelerometers are included in the model (Fig. 2). Solid bricks are used to model the plate, the load cell and the piezoelectric actuators; in particular, specific solid elements for electromechanical coupling problems are used to model the PZT patches. The accelerometers (2 grams) are represented as lumped masses. The geometry of the load cell is modeled considering that the experimental measurements are influenced by half of its mass and all its rotary inertia.

The resulting sequence of modal shapes is the same as

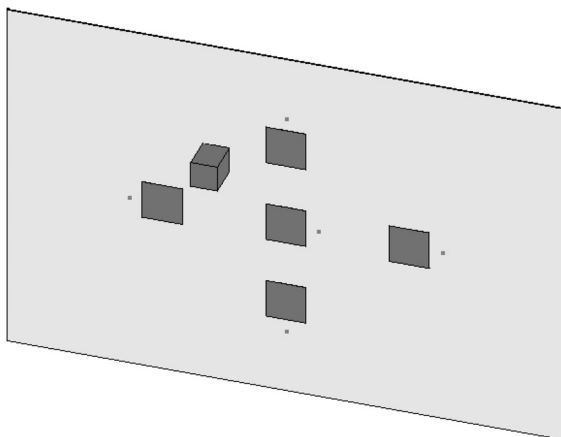


Fig. 2. Finite-elements model of the system: geometry.

Table 1  
Natural frequencies

	Modal shape	FEM model natural frequencies [Hz]	Experimental natural frequencies [Hz]
Mode 1	(1,1)	84.5	83.5
Mode 2	(2,1)	135.3	162.6
Mode 3	(1,2)	211.4	190.7

the experimental tests, even if the frequency values don't agree exactly (Table 1). Reasonable differences between numerical and experimental values could be due to non-uniform real boundary conditions and possible shape-errors of the plate. Introducing PZT patches and accelerometers in the model produces the same sequence of modes with a reduction in the natural frequencies, as verified in the experimental case.

### 4. The control system

Five piezoelectric PZT patches ( $25 \times 25 \times 0.2$  mm) are glued to the dry surface of the plate. Their position has been optimized by numerical calculations of the deformation energy of the plate in the case of empty tank. Each accelerometer is positioned close to an actuator, realizing a nearly-collocated control channel (Fig. 3). Experimental control tests are performed using a feed-forward adaptive control based on a filtered-x last mean squared (LMS) technique, as described in [7]. Such an algorithm has been implemented on a real-time digital controller working in a Matlab-Simulink environment.

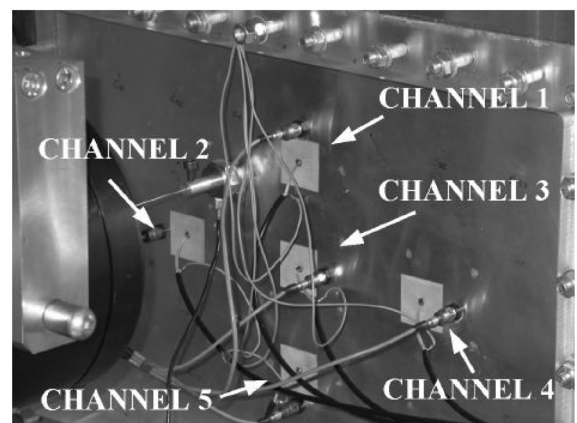


Fig. 3. The control channels.

Table 2  
Control effectiveness (dB) on the acceleration signals for the first three modes

	Empty tank Reductions (dB)			Water-filled tank Reductions (dB)		
	1st mode	2nd mode	3rd mode	1st mode	2nd mode	3rd mode
Channel 1	-17.77	-27.40	-17.07	-5.50	-10.45	-3.34
Channel 2	-14.22	-39.05	+0.96	-4.86	-9.19	-0.83
Channel 3	-38.15	-20.55	-18.24	-3.51	-3.58	-1.76
Channel 4	-23.28	-45.38	+3.03	-4.43	-19.68	-1.74
Channel 5	-23.28	-29.67	-14.15	-4.86	-3.62	-1.31

## 5. Feedforward control: experimental results

A filtered-x LMS algorithm is applied to the system, in presence of sinusoidal perturbation, with SISO approach.

The effectiveness of active control on the first three modes is investigated, with and without fluid. Table 2 shows the reduction (sign  $-$ ) of the peak of the FFT spectrum of the acceleration signals, expressed in dB.

### 5.1. Empty tank

In the case of absence of fluid, the SISO approach has a good effect on the first mode, with a particularly important reduction on Channel 3, which is optimally-located to control mode (1,1). The other channels show a good effectiveness, with an unsymmetrical behavior due to the eccentric load cell. A reduction of the acceleration can be observed on the second mode (2,1) especially on the optimally-located channels 2 and 4. Finally, actuators 1 and 5 optimally control the modal shape (1,2) of the third mode, while actuators 2 and 4 (located on nodal lines) produce a little amplification of the accelerations.

### 5.2. Water-filled tank

In the case of completely water-filled tank, gravity and sloshing effects do not give significant effects on the structural modes [1,2]. In fact, sloshing frequencies can be found at much lower frequencies than the first structural mode [8].

The effectiveness of control on the first mode seems to be uniformly distributed on all the five control channels. Regarding the second and third modes, each PZT patch of the optimally-located couples of actuators (number 2 and 4 for mode shape (2,1); number 1 and 5 for mode shape (1,2)) presents different values of reduction. Possible reasons for the reduced effectiveness of control with water are related to the deformations due to the presence of fluid and to the small dimensions of the actuators.

## 6. Conclusions

The modal analyses of the rectangular plate for different water levels show that the fluid gives a significant reduction of the natural frequencies. After attaching piezoactuators and sensors, the sequence of modes doesn't change. In absence of water, the same modal shapes also appear if the dynamics of the system is simulated using a 3D finite-elements model.

Applying feedforward control, good reductions (up to 38 dB on Channel 3 for the first mode) are obtained for sinusoidal excitation in case of empty tank, and the particular effectiveness of the optimally-located actuators is apparent.

Introducing water, lower effects (up to 5.5 dB for the first mode) are observed, but they are more uniformly distributed on all the control channels.

Further experimental investigations will be performed, even in presence of white-noise excitation. Possible shape-errors of the plate and non-uniform real boundary conditions will be included in the finite-elements model, even in case of fluid-structure interaction.

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