Transfer operator based on diffuse interpolation and energy conservation for damage materials

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Abstract

The work presented herein consists in an adaptive remeshing technique dealing with damage material. More particular details concerning the development of a transfer operator are addressed. This operator is based on diffuse interpolation and ensures energy conservation between the old discretized domain and the new one.

Keywords: Damage material; Remeshing technique; Transfer operator; Diffuse interpolation

1. Introduction

For a large class of problems (forming processes, localization problems, hightly heterogeneous loadings, ...), the quality and predictivity of the numerical simulation require the remeshing of the computational domain into the optimal discretization configuration.

Such adaptive techniques are based on the development of effective error estimators and mesh refinement procedures but also on reliable transfer operators allowing to continue the computation on a new discretization. Several important aspects of the transfer operator have to be adressed [1], more particularly, the consistency with constitutive equations and the conservation of the equilibrium equation.

The work presented herein consists mainly in the development of a transfer operator dealing with nonlinear materials and, more precisely, damage materials. The transfer operator ensures the consistency with the constitutive equations, the equilibrium equations and preserves energetic quantities related to the damage state of the structure: dissipated energy during the loading and strain energy.

We first present the isotropic damage model used in this work. Then, we develop the main aspects of the transfer operator proposed and, finally, we give some numerical results and conclusions.

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Computational Fluid and Solid Mechanics 2005
K.J. Bathe (Editor)

2. Damage constitutive model

The model presented here is an isotropic damage model with isotropic 'hardening'. Such a model is assumed to represent the progressive nucleation of micro-cracks in the bulk material distributed of more or less random orientation which can be considered as inducing the damage in roughly isotropic way. The internal variables of the model are the (forth order) compliance tensor denoted as: $\bar{D}$ and the hardening variable denoted as $\bar{\xi}$ [2].

The main ingredients of the construction of the model are summarized in Table 1. In Table 1, $\bar{Y}$ is the thermodynamic force associated with the compliance $\bar{D}$. All the relations can be deduced by appealing to the thermodynamics principles and the principle of maximum dissipation.

3. Transfer operator

Some general aspects of the transfer operator proposed in this work are addressed in this section. The transfer operator developed herein ensures that the fields associated to the new discretization satisfy the local equilibrium equations, the damage criterion and conserves between the old and new meshes energy quantities: dissipated and strain energies.

The first step of the transfer consists in constructing, by using diffuse interpolation [3], the internal variables fields on the new mesh. The field of the increment of

Table 1
Main ingredients of the damage model

Constitutive damage model	
State variables/dual variables	$(\bar{\varepsilon},\ \sigma)$, $(\bar{\mathbf{D}},\ \bar{\mathbf{Y}})$, $(\bar{\xi},\ \bar{q}))$
Helmholtz free energy	$\bar{\psi}(\bar{\varepsilon},\ \bar{\mathbf{D}},\ \bar{\xi}) = \frac{1}{2}\bar{\varepsilon} : \bar{\mathbf{D}}^{-1} : \bar{\varepsilon} + \bar{\Xi}(\bar{\xi})$
Damage function	$\bar{\phi}(\sigma,\ \bar{q}) = \sqrt{\underbrace{\sigma : \mathbf{D}^e : \sigma}_{\Vert\sigma\Vert_{\mathbf{D}^e}}} \ - \frac{1}{\sqrt{E}}(\bar{\sigma}_f - \bar{q}) \leq 0$
State equations	$\sigma = \bar{\mathbf{D}}^{-1} : \bar{\varepsilon}$ and $\bar{q} = -\frac{d}{d\bar{\xi}}\bar{\Xi}(\bar{\xi})$
Evolution equations	
Dissipation	$0 < D = \frac{1}{2}\bar{\varepsilon} : \dot{\bar{\mathbf{D}}}^{-1} : \bar{\varepsilon} + \bar{q}\dot{\bar{\xi}}$
Internal variables evolution	$\dot{\bar{\mathbf{D}}} = \dot{\bar{\gamma}}\frac{\partial\bar{\phi}}{\partial\sigma} \otimes \frac{\partial\bar{\phi}}{\partial\sigma}\frac{1}{\Vert\sigma\Vert_{\mathbf{D}^e}}$; $\dot{\bar{\xi}} = \dot{\bar{\gamma}}\frac{\partial\bar{\phi}}{\partial\bar{q}}$
Stress evolution	$\dot{\sigma} = \mathbf{C}^{\mathrm{ed}}\ \dot{\epsilon}$
	$C^{ed} = \bar{\mathbf{D}}^{-1}$ if $\dot{\bar{\gamma}} = 0$
	$= \bar{\mathbf{D}}^{-1} - \frac{\left(\bar{\mathbf{D}}^{-1}:\frac{\partial\bar{\phi}}{\partial\sigma}\right) \otimes \left(\bar{\mathbf{D}}^{-1}:\frac{\partial\bar{\phi}}{\partial\sigma}\right)}{\frac{\partial\bar{\phi}}{\partial\sigma}:\bar{\mathbf{D}}^{-1}:\frac{\partial\bar{\phi}}{\partial\sigma} + \bar{K}\left(\frac{\partial\bar{\phi}}{\partial\bar{q}}\right)^2}$ if $\dot{\bar{\gamma}} > 0$

those variables is reconstructed at the same time. Then, the fields $\bar{\xi}$ and $\Delta\bar{\xi}$ (where $\Delta\bar{\xi} = \bar{\xi}_n - \bar{\xi}_{n-1}$, n being the time step at which remeshing is decided) are known all over the new computational domain.

Then, internal variables are renormalized in order to ensure the conservation of the dissipated energy. Moreover, in order to control numerical diffusion, elements where the variable $\bar{\xi}$ is lower than a threshold defined by statistical criteria are not considered as damaged.

This first step allows to define two different zones of the computational domain:

- zone 1: at time t_n, internal variables are evoluating, the material is damaging and $\bar{\phi} = 0$;
- zone 2: at time t_n, internal variables are not evoluating, the material is loading or unloading elastically and $\bar{\phi} \leq 0$.

The second step of the transfer consists in reconstructing the stress field on the new mesh satisfying the damage criterion $\bar{\phi} \leq 0$. At the same time, the conservation of the strain energy is ensured.

The strategy for reconstructing the stress field differs if we consider a point either in zone 1 or in zone 2.

For the points in zone 1 (in this case, integration points of the new computational domain), the reconstruction of the stress field is carried out by appealing to diffuse interpolation to construct, from the discrete field known on the old discretization, a new field of arbitrary order continuity (in our case C^1) satisfying the admissibility condition that is $\bar{\phi} = 0$. Noting that the admissibility can be rewritten as a quadratic constraint, such a problem results in the resolution of a quadratic optimization problem with quadratic constraints [4].

For the points in zone 2, the reconstruction of the stress field is carried out by using a simple diffuse interpolation, which results in the resolution of a simple quadratic optimization problem without constraint. In this zone, the conservation of the strain energy is ensured by renormalizing the stress field obtained. Such a procedure does not ensure the stress admissibility $\bar{\phi} \leq 0$. The admissibility is guaranteed by constructing a cell automat: the strain energy remaining constant, the cell automat redistributes stresses by loading and unloading in zone 1 ensuring that, at each point, $\bar{\phi} \leq 0$.

4. Numerical results

We present here some numerical results obtained by using the transfer operator presented here above. The test considered is a traction test on a notched beam. Two different regular discretization are considered, a coarse and a finer one. The damage model presented has been implemented in the Finite Element code *FEAP* developed by Professor R.L. Taylor at Berkeley, California.

Fields are reconstructed on the fine discretization from the discrete fields known on the coarse mesh. In order to evaluate the quality of the reconstructed field (here the field $\bar{\xi}$ of the hardening variable), we also give the field computed by the finite element code considering the fine mesh from the beginning of the computation.

The results are given in Figs 1–3.

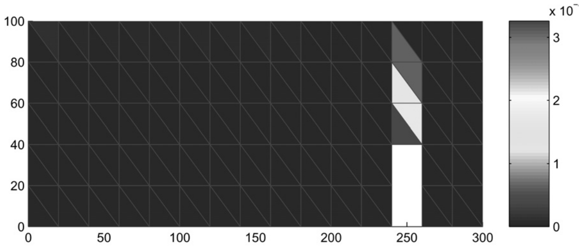

Fig. 1. Damage (variable $\bar{\xi}$) on the old mesh.

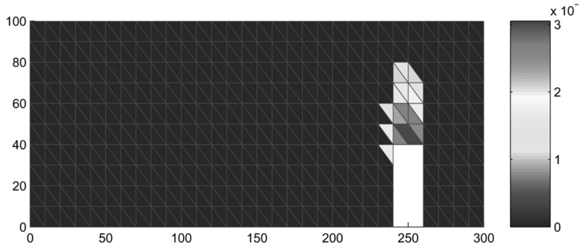

Fig. 2. Damage variable $\bar{\xi}$ transferred on the new mesh.

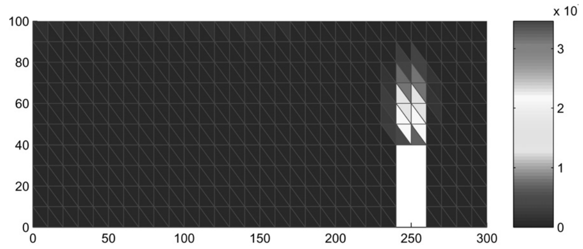

Fig. 3. Damage variable $\bar{\xi}$ obtained by direct computation on the new mesh.

The results given both by the transfer operator and the direct computation are quite similar, the damage zone is well captured and numerical diffusion is limited.

5. Conclusion

The work presented here deals mainly with the development of a transfer operator for structures involving damage materials. The operator proposed here for the reconstruction of fields on a new discretization is based on diffuse interpolation. The key point of the operator proposed is to ensure:

- local equilibrium,
- stress admissibility,
- preservation of energetic quantities defining the damage state of the structure: strain energy and dissipated energy.

A refinement procedure is being developed in accordance with this field transfer operator to improve the results of the remeshing technique.

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