Finite-element time-domain simulations of bridge aeroelasticity: implementation and profiling

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Abstract

The aeroelastic self-excited load model via indicial functions has reached sufficient theoretical maturity in the academic realm. In order to be used in the real practice of bridge design, a reliable procedure must be provided to obtain indicial functions and an efficient implementation of the convolution integrals into structural analysis programs is required. The latter problem is addressed in the present paper, where an optimized model with fading aeroelastic-memory is developed and implemented, and a first criterion to evaluate the optimal size of the memory is proposed. A substantial reduction of the computing time is obtained without compromising the accuracy of the analyses.

Keywords: Bridge aerodynamics; Aeroelasticity; Indicial functions; Structural dynamics; Finite elements; Computing efficiency

1. Introduction

Even more than in the past, bridge aerodynamics is a crucial topic, due both to the increase of the span length and to the trend towards very light and architectonically extravagant structural solutions for smaller bridges. Concerning wind load modeling, in addition to the static (mean) action, two dynamic load mechanisms are of prime importance: the gust excitation due to turbulence of the wind flow, and self-excited aeroelastic forces, which can result in instability phenomena such as bridge flutter. The latter aspect is addressed in the present paper. Mixed time-frequency domain models for aeroelastic loads are well established. Nevertheless, the pure time domain approach through indicial functions offers several advantages [1] and is now theoretically mature [2]. Here, some implementation details are presented and an evaluation of the computational performance is carried out.

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2. Self-excited aeroelastic loads

2.1. The indicial function model

Several (slightly different) expressions of the indicial function model have appeared after its introduction by Scanlan et al. [3]. Here, the formulas proposed by Costa [4] are considered. The lift force and twisting moment acting on the unit-span bridge-deck cross-section (see Fig. 1) at time t are given by

$$L(t) = qBC'_L\left(\Phi_{L\dot{y}}(0)\frac{\dot{y}(t)}{U} + \Phi_{L\alpha}(0)\alpha(t) + \int_0^t \left[\dot{\Phi}_{L\dot{y}}(t-\tau)\frac{\dot{y}(\tau)}{U} + \dot{\Phi}_{L\alpha}(t-\tau)\alpha(\tau)\right]d\tau\right)$$

$$M(t) = qB^2C'_M\left(\Phi_{M\dot{y}}(0)\frac{\dot{y}(t)}{U} + \Phi_{M\alpha}(0)\alpha(t) + \int_0^t \left[\dot{\Phi}_{M\dot{y}}(t-\tau)\frac{\dot{y}(\tau)}{U} + \dot{\Phi}_{M\alpha}(t-\tau)\alpha(\tau)\right]d\tau\right) \quad (1)$$

where $q = 1/2\rho U^2$ = kinetic pressure; ρ = air density; U = mean wind velocity; B = bridge-deck width; C'_L

and C'_M = derivatives of the static lift and moment coefficients with respect to the angle of attack evaluated in the static (mean) position; y and α = vertical and torsional displacements; $\Phi_{Xx}(t)$ (with $X = L,M$; $x = \dot{y}$, α) = indicial functions describing the time evolution of the action X due to a step change of x at $t = 0$. Each dot denotes a derivation with respect to the time t.

2.2. Identification of indicial function

Although some attempts have been made to directly measure indicial functions in experimental tests [5], and Computational Fluid Dynamics seems to offer a promising tool [6], the most feasible procedure consists to date in choosing a parametric analytical expression of indicial functions and in identifying the parameters from quantities measured in the wind tunnel. Here, exponential filters [7] are considered:

$$\Phi_{Xx}(t) = 1 - \sum_{j=1}^{N_{Xx}} a_j^{Xx} \exp\left(-b_j^{Xx} \frac{2U}{B} t\right)$$
$$X = L, M \quad x = \dot{y}, \alpha \tag{2}$$

The dimensionless parameters a_j^{Xx} and b_j^{Xx} describe, for each term, the 'amplitude' and the 'decay' respectively, whereas the number of terms N_{Xx} accounts for the accuracy of the approximation of Φ_{Xx}. It is worth to notice that the coefficients b_j^{Xx} must be positive, as the forces arising from the step change tend to a stationary value as $t \to \infty$.

A reliable procedure for the identification of indicial function coefficients has been developed in Zahlten et al. [1].

3. Computational efficiency

3.1. Advisability of the optimization

Time-domain aeroelastic simulations of bridges are very expensive in terms of computational resources, since:

- the time step used in the simulations must be kept small to avoid numerical damping and to accurately evaluate the convolution integrals of Eqs. (1);
- iterations must be performed at each time step, in order to take into account structural non-linearities, if necessary;
- several simulations must be carried out, varying the wind speed and comparing the time histories, until the critical condition (i.e. unstable oscillations) is reached;
- the simulations must be long enough in order to accurately observe the critical condition;
- at each time step the convolution integrals become longer and longer (the time to compute the loads increases therefore quadratically with the number of time-steps).

For systematic studies as well as in the preliminary design phase of a bridge, strategies to save computing time would be very useful. One approach, discussed in Salvatori et al. [8], is to reduce the complexity of the FE structural model (and of the number of cross-sections where the convolution integrals are calculated). Here, a strategy to reduce the length of the integration interval in Eqs. (1) is presented.

3.2. Finite 'aeroelastic memory'

The time-derivatives of indicial functions that appear in the convolution integrals of Eqs. (1), vanish for $t \to \infty$. In particular, for a T_{mem} 'sufficiently large', the following implications holds:

$$t > T_{\mathrm{mem}} \quad \Rightarrow \quad \frac{|\dot{\Phi}_{Xx}(t)|}{\max\limits_{\tau \geq 0} |\dot{\Phi}_{Xx}(\tau)|} \ll 1 \quad \Rightarrow$$
$$\int_0^t \dot{\Phi}_{Xx}(t-\tau)x(\tau)d\tau \simeq \int_{t-T_{\mathrm{mem}}}^t \dot{\Phi}_{Xx}(t-\tau)x(\tau)d\tau \tag{3}$$

Equations (1) can be therefore rewritten as

$$L(t) = qBC'_L\Bigg(\Phi_{L\dot{y}}(0)\frac{\dot{y}(t)}{U} + \Phi_{L\alpha}(0)\alpha(t) + \int_{t-T_{\mathrm{mem}}}^t \left[\dot{\Phi}_{L\dot{y}}(t-\tau)\frac{\dot{y}(\tau)}{U} + \dot{\Phi}_{L\alpha}(t-\tau)\alpha(\tau)\right]d\tau\Bigg)$$
$$M(t) = qB^2C'_M\Bigg(\Phi_{M\dot{y}}(0)\frac{\dot{y}(t)}{U} + \Phi_{M\alpha}(0)\alpha(t) + \int_{t-T_{\mathrm{mem}}}^t \left[\dot{\Phi}_{M\dot{y}}(t-\tau)\frac{\dot{y}(\tau)}{U} + \dot{\Phi}_{M\alpha}(t-\tau)\alpha(\tau)\right]d\tau\Bigg) \tag{4}$$

A finite memory approach with an incremental formulation was proposed by Borri et al. [9] but no investigation of the computing time involved was performed and the choice of the length of the memory was made in a safe but arbitrary way. Here the question of the memory length is systematically investigated, weighting computational performance and accuracy. Moreover, the adopted expressions of the load (Eqs. (1)) avoid the incremental formulation.

In order to find a general criterion for the choice of T_{mem}, given a set of indicial functions, the scalar index

$$R(T_{\text{mem}}) = \max_{\substack{X=L,M \\ x=\dot{y},\alpha}} \left(\frac{\left|\dot{\Phi}_{Xx}(T_{\text{mem}})\right|}{\max_{t \geq 0} \left|\dot{\Phi}_{Xx}(t)\right|} \right) \tag{5}$$

is proposed, which accounts for the relative size of the neglected tail of the indicial function that converges 'more slowly'.

4. Implementation

A specific code for the study of bridge aeroelasticity has been developed [9], which integrates a pre-processor for the parametric generation of the bridge FE model, a multi-correlated wind velocity field generator, an FE solver for non-linear dynamic problems (Newmark integration, Newton-Raphson iterations at each time-step), and a post-processor.

The aeroelastic forces are assembled through special one-node elements. Equations (4) are integrated with the left rectangular rule (trapezoidal or Simpson's rule give no accuracy improvements and involve a load dependence on the current step, i.e. stepwise iteration also for linear structures).

5. Numerical examples

5.1. 2DoF system

As first test, flutter simulations are performed on the 2DoF system sketched in Fig. 1 (k_y = 5180 (N/m)/m; k_α = 100.6 (N·m)/m; ξ_y = 0.18%; ξ_α = 0.28%; m_y = 3.81 kg/m; m_α = 0.037 (kg·m^2)/m; B = 0.376 m; depth = l = 0.92 m) with the aerodynamic characteristics of the streamlined rectangular cross-section with dimension ratio B/D = 12.5 (experimentally tested by Righi [10]). The critical flutter condition is evaluated both with an infinite aeroelastic memory and with several values of the finite memory. A time history of ten seconds is simulated and the program profiled. The timing result are obtained on an AMD® Athlon™ XP 1700+ processor (1466 MHz). Less systematic tests on different processors have produced comparable performance ratios. Each test has been executed at least three times. The results in terms of critical conditions and average timings are presented in Table 1.

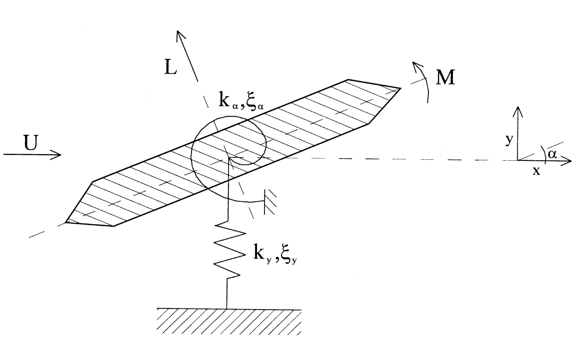

Fig. 1. Two-degree-of-freedom cross-sectional model.

An abrupt reduction in the total computing time is observed when passing from an infinite to a finite aeroelastic memory. Moreover, to consider a memory longer than that corresponding to R = 10^{-3} does not bring any accuracy improvement. This suggests to identify a criterion to decide the optimal memory length. Further tests on different cross sections will lead to more general and reliable criteria. Time histories of motion obtained with infinite and finite memory compare very well as shown in Fig. 2.

5.2. Suspension bridge simulation

As second example, the aeroelastic behavior of the Bosporus suspension bridge is simulated. Even if a rather simplified FE model is considered (with a reduced number of loaded cross-sections), the aeroelastic analyses last 3–4 hours when the infinite memory is employed, whereas the finite memory allows a reduction in the computing time of almost a factor three, without noticeable loss of accuracy. The details are omitted here for lack of space.

6. Concluding remarks

In the perspective of the practical applicability of the indicial function model, an efficient implementation is presented together with a proposal for a general criterion to estimate the optimal size of the aeroelastic memory that greatly reduces the computational time without loss in accuracy. The ongoing research is considering complete sets of cross-sections, also at different angles of attack.

Turbulence analyses and Monte-Carlo simulations on the along-span characteristics are further steps, which will profit of the computational efficiency of the fading memory formulation.

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Table 1
Simulation results and timings for various aeroelastic memory lengths

R	T_{mem} (s)	U_{cr} (m/s)	f_{cr} (Hz)	CT_{Tot}[a] (s)	CT_{CI}[b] (s)	CT_{FE}[c] (s)
0	∞	15.25	7.35	1201	814	387
10^{-4}	0.171	15.25	7.35	348	25	323
10^{-3}	0.128	15.26	7.35	346	23	323
10^{-2}	0.085	15.22	7.36	342	20	322
10^{-1}	0.043	15.03	7.39	340	18	322
0.5	0.013	16.04	7.42	336	14	322

[a] Total computing-time; [b] convolution-integral computing-time; [c] finite-element solver computing-time $CT_{FE} = CT_{Tot} - CT_{CI}$.

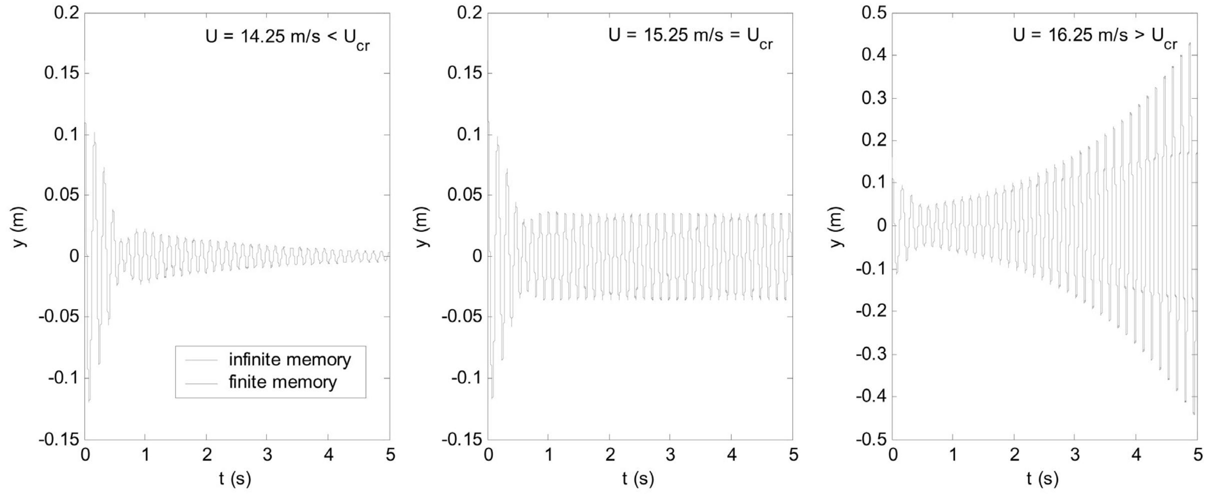

Fig. 2. Oscillation of the two-degree-of-freedom system for under-critical, critical and super-critical wind velocities: comparison of the infinite and finite memory models.

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References

[1] Zahlten W, Salvatori L, Borri C. Indicial functions or flutter derivatives: an evaluation for time domain simulations. Proc IN-VENTO 2004, Reggio Calabria, Italy.
[2] Chen X, Matsumoto M, Kareem A. Time domain flutter and buffeting response analysis of bridges. J Eng Mech 2000;126(1):17–26.
[3] Scanlan RH, Béliveau J-G, Budlong K. Indicial aerodynamic functions for bridge decks. J Eng Mech Div 1974;100:657–672.
[4] Costa C. Time-domain models in bridge deck aeroelasticity. PhD thesis, Univ Florence, Italy, 2004.
[5] Caracoglia L, Jones NP. A methodology for the experimental extraction of indicial functions for streamlined and bluff deck sections. J Wind Eng Ind Aerodyn 2003;91:609–636.
[6] Larsen A. Aerodynamic derivatives from simulated indicial functions. Proc IC-WE 2003, Lubbock, Texas, 2003.
[7] Höffer R. Stationäre und instationäre Modelle zur Zeitbereichssimulation von Windkräften an linienförmigen Bauwerken. PhD thesis, Univ Bochum, Germany, 1997.
[8] Salvatori L, Spinelli P. Influence of structural non-linearities on the response of suspended bridges under wind excitation: numerical simulations on simplified structures. Proc IN-VENTO 2004, Reggio Calabria, Italy.

[9] Borri C, Costa C, Zahlten W. Non-stationary flow forces for numerical simulation of aeroelastic instability of bridge decks. Comput Struct 2002;80:1071–1079.

[10] Righi M. Aeroelastic stability of long span suspended bridges: flutter mechanism on rectangular cylinders in smooth and turbulent flow. PhD thesis, Univ Florence, 2003.