Medium frequency computations of structures including interface uncertainties

C. Blanzé*, P. Rouch

LMT Cachan, ENS de Cachan/CNRS/Paris VI, 61, av. du Pt Wilson, F94235 Cachan Cedex, France

Abstract

This paper proposes efficient techniques to obtain effective quantities when confronted to complex structures in medium frequency vibrations including stochastic parameters such as interface parameters. The first ingredient is the use of a dedicated approach to compute the medium-frequency problem: the Variational Theory of Complex Rays (VTCR). Using two-scale shape functions that satisfy the dynamic equation and the constitutive relation, the VTCR can be seen as a power balance on the different interfaces between sub-structures. The second ingredient is the use of the Polynomial Chaos Expansion (PCE) to compute the random response. Since the only uncertain parameters (in this application: the complex connection stiffness) are those that appear in the interface equations, this approach lead to very small computation cost.

Keywords: Medium frequency vibrations; Stochastic interface; Polynomial chaos; VTCR

1. Introduction

Today, the main numerical modeling techniques for the analysis of medium-frequency vibrations are all based on finite element or boundary element approaches. In order to represent small-wavelength phenomena in complex structures such as car chassis, satellites or ships, these techniques require a huge number of degrees of freedom (at least seven elements per wavelength are required to represent oscillating solutions). In addition, the solution obtained is highly sensitive to material properties and boundary conditions. Moreover, the damping is often localized in the different structure connections. Its measure is very difficult and highly uncertain. In order to obtain the stochastic responses of the structure, one needs to carry out a large number of costly computations. The use of high-frequency approaches such as the Statistical Energy Analysis (SEA) [1] or any of its improvements does not appear to be suitable for medium-frequency vibrations: the vibrational behavior becomes too global and, in general, the coupling loss factor cannot be calculated in a predictive way. This paper proposes efficient techniques to obtain effective quantities when confronted to complex structures including stochastic parameters such as interface parameters. The application presented concerns a 3D plates assembly including two uncertain elastically bonded joints.

* Corresponding author. Tel.: +33 1 47 40 21 86; Fax: +33 1 47 40 22 40; E-mail: claude.blanze@lmt.ens-cachan.fr

2. Basic aspects of the VTCR

2.1. The reference problem

Here, in order to simplify the presentation, the problem will be formulated for an assembly of only two substructures, but this can easily be generalized to an assembly of n substructures. For the purpose of illustration, let us consider the steady-state vibrations of two thin, homogeneous, isotropic and elastic Kirchhoff-Love plates. Classically, all quantities are defined in the complex domain: an amplitude $\underline{Q}(\underline{X})$ is associated with $\underline{Q}(\underline{X}).\exp(i\omega t)$.

Let S_1 and S_2 be two plates, ∂S_1 and ∂S_2 their respectively boundaries solicited harmonically at a fixed angular frequency ω and Γ_{12} the perfect interface between S_1 and S_2. The quantities of interest are the deflections w_l and the moments $\mathbf{M}_l$. For S_l, let us introduce the space $\mathcal{S}_{ad}^{l}$ of the deflection–moment pairs $s_l = (w_l, \mathbf{M}_l)$ defined on S_l such that:

Computational Fluid and Solid Mechanics 2005
K.J. Bathe (Editor)

$$s_l \in \mathcal{S}_{ad}^l \quad \Leftrightarrow \quad \begin{cases} s_l \in U \times \mathcal{S} \quad \text{(set of finite - energy fields)} \\ \Delta\Delta w_l - k_l^4 w_l = 0 \quad \text{on } S_l \\ \mathbf{M}_l = \frac{2h_l^3}{3}(1 + i\eta_l)\, \mathbf{K}_{PSl}\chi(w_l) \\ \quad \text{for} \quad l = 1,2 \quad \text{with} \quad k_l^4 = \frac{3\rho_l\omega^2(1-\nu_l^2)}{(1+i\eta_l)E_l h_l^2} \end{cases} \tag{1}$$

where k_l is the wave number and ρ_l, ν_l, η_l, E_l and $2h_l$ designate respectively the density, Poisson's ratio, structural damping coefficient, Young's modulus and thickness of plate S_l. $\mathbf{K}_{PSl}$ is Hooke's tensor for plane stress and χ is the curvature operator. The fields of $\mathcal{S}_{ad}^l$ are admissible in the sense that they verify both the local equilibrium and the constitutive relation exactly.

2.2 The variational formulation associated with the VTCR

The VTCR is primarily a global formulation of the boundary conditions and transmission conditions in terms of both displacements and forces across Γ_{12} [2,3].

In practice, the VTCR procedure consists of two stages: first, admissible fields are built; then, the boundary conditions are taken into account through the discretization of the variational formulation associated with a set of admissible fields chosen among the elements of $\mathcal{S}_{ad}$. Let us define a subset of $\mathcal{S}_{ad}^l$. The VTCR uses two-scale approximations. For example, in the case of a $0th$-order complex ray, the solution is described by the superposition of modes that can be written in the following manner:

$$W(\underline{X},\underline{P}) = a(\underline{P})\ \exp\left(\frac{\eta}{4}\sqrt{\omega}\underline{P}\cdot\underline{X}\right)\ \exp\left(i\sqrt{\omega}\underline{P}\cdot\underline{X}\right) = a(\underline{P})w_e(\underline{X},\underline{P}) \tag{2}$$

$e^{i\sqrt{\omega}\underline{P}.\underline{X}}$ represents the 'fast' part of the field, which is defined explicitly. $\underline{P}$ designates a vector characterizing the local vibration mode and chosen so that the mode verifies the dynamic equations and $a(\underline{P})$ is the complex magnitude of the elementary complex ray $w_e(\underline{X},\underline{P})$.

There are many possible choices depending on the zone whose vibrations are particularly sought; the VTCR distinguishes among the interior zone, the edge zone and the corner zone. In order to take into account the admissibility relation (Eq. (1)), the locus of the end of the admissible wave vector $\underline{P}$ for the interior zone have to belong to a circle C. This curve is a characteristic of the material. By following the circular path, each direction of the plate is taken into account. The definitions of corner and edge modes are very similar. For the interior zone, one takes into account all admissible ray directions. The solution w^{sol} is written as:

$$w^{sol}(\underline{X}) = \int_C W(\underline{X},\underline{P})\, ds \tag{3}$$

The integrals are calculated along curve C. Clearly, w^{sol} depends on the generalized amplitude $a(\underline{P})$ defined on C. All that is needed in order to derive approximations from the VTCR is the definition of subspace $\mathcal{S}_{ad}^{lh}$ from $\mathcal{S}_{ad}^l$. In practice, one discretizes curve C into N elements that can be of different sizes (on Fig. 1, C is discretized into eight elements of the same size). The amplitude $a(\underline{P})$ is considered to be constant along angular sectors a_i^h. Then:

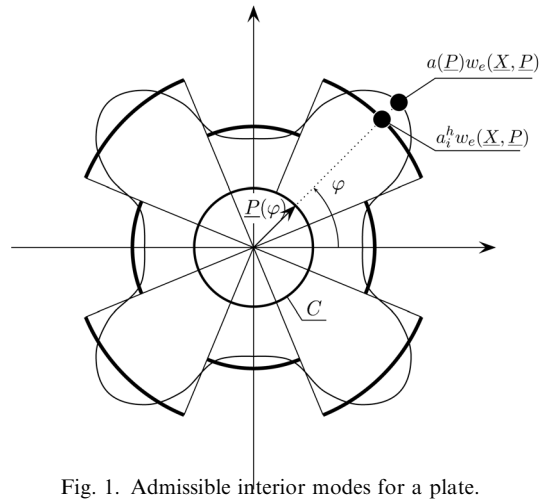


Fig. 1. Admissible interior modes for a plate.

$$w^h(X) = \sum_{i=1}^{n} a_i^h \int_{C_i} w_e(\underline{X},\underline{P})\, ds = \sum_{i=1}^{n} a_i^h w_i^h(X) \tag{4}$$

In the case of an assembly of two substructures S_1 and S_2, the variational formulation leads to a system of equations of finite dimension in the complex domain:

$$\mathbf{Ka} = \mathbf{f} \tag{5}$$

$\mathbf{K} = \mathbf{E_D^h} + \mathbf{Z_h}$, where $\mathbf{E_D^h}$ is the symmetric, positive definite damping matrix associated with the dissipated power, and $\mathbf{Z}_\mathrm{h}$ is the matrix associated with a bilinear form defined such that $\mathbf{Z}_\mathrm{h}^{T*} = \mathbf{Z}_\mathrm{h}$. $\mathbf{f}$ is the vector associated with the linear form and $\mathbf{a}$ is the vector of the unknown amplitudes.

3. Modeling of the interfaces

Today, the modeling of beams, plates, shells and their combination no longer presents difficulties for the VTCR, but the connections among these structures can be improved in order to take into account more realistic types of behavior, such as damping localized in the

different joints connecting the substructures. Since the wavelengths of the vibrational phenomena in medium-frequency are relatively large compared with the distance between two consecutive rivets or two welded points, we decided to introduce these by using distributed interface parameters such as damping and stiffness coefficients, identified on generic assemblies.

3.1. The interface model

This local behavior can be modeled by interface relations that represent elastic and/or dissipative joint parameters. Each interface is a mechanical entity whose specific behavior depends on the type of connection. Many different connection types can be modeled by this approach:

$$-\underline{n_1}\mathbf{M}_1\underline{n_1} + \underline{n_2}\mathbf{M}_2\underline{n_2} = 0 \qquad \text{Equilibrium} \qquad (6)$$

$$\frac{1}{2}(\underline{n_1}\mathbf{M}_1\underline{n_1} + \underline{n_2}\mathbf{M}_2\underline{n_2}) = -k_c(1 + i\eta_c)$$
$$(w_{1,n_1} + w_{2,n_2}) \qquad \text{Behavior} \qquad (7)$$

where k_c is the bending stiffness coefficient and η_c the damping coefficient.

Here, we are dealing with problems of structural assemblies in which the component's behavior and the external loads are deterministic: the randomness of the response comes from the random behavior of the connections. The system's parameters, calibrated using experimental data, are modeled as random variables or processes that are assumed to be properly represented by a set of random variables $\{\xi(\theta)\}$, where θ belongs to the space of random events Ω. More precisely, the bending stiffness coefficient k_c, which may be uncertain, is modeled by a random variable.

3.2. Discretization in random space: polynomial chaos expansion (PCE)

The stochastic interface parameters lead to a discretized linear system similar to Eq. (5):

$$\mathbf{K}(\theta)\,\mathbf{a}(\theta) = \mathbf{f} \qquad (8)$$

The generalized variables $\mathbf{a}(\theta)$ can be expressed formally as a nonlinear functional of the set $\{\xi(\theta)\}$ used to represent the material's stochastic property. It has been shown [4] that this functional dependence can be expanded in terms of polynomial chaos. Then, the truncated PCE [5] of the response takes the form:

$$\mathbf{a}(\theta) = \sum_{i=0}^{P} \mathbf{a}_i \Psi_i(\theta) \qquad (9)$$

where $\{\Psi_i(\theta)\}$ are polynomials in the Gaussian random variables $\{\xi_i\}$, which can be shown to form a complete basis of the Hilbert space of second-order random variables. The number of polynomials P depends on the order p of the PCE and on the number L of stochastic parameters. Let us assume that the material parameters are constant along an interface. If this were not the case, a Karhunen-Loéve expansion could easily be used to represent the spatial randomness of the interface's characteristics [6].

The random character of a joint's parameter $\alpha(\theta)$ is made explicit by its argument θ. The stochastic parameter $\alpha(\theta)$ is represented by

$$\alpha(\theta) = \bar{\alpha}(1 + \delta\xi(\theta)) \qquad (10)$$

where $\bar{\alpha}$ is the mathematical expectation of $\alpha(\theta)$, δ the coefficient of variation (standard deviation divided by the expectation) and $\xi(\theta)$ a standard normal random variable: $\xi(\theta) \sim N(0,\ 1)$. For non-Gaussian material properties, the PCE is used to represent the stochastic material's parameters: the case of a Gaussian process is a particular case which was chosen for the sake of simplicity [7].

The system (Eq. (8)) can be rewritten as:

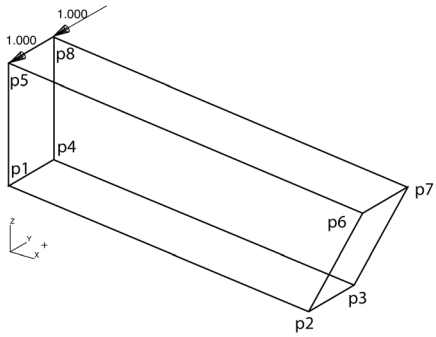
$$\sum_{j=0}^{P}\sum_{i=0}^{L} c_{ijk}\mathbf{K}_i\mathbf{a}_j = \delta_{0k}\mathbf{f} \quad k = 0,\ 1, \ldots P, \qquad (11)$$

where the coefficients c_{ijk} denote $\langle \xi_i \Psi_j(\theta)\Psi_k(\theta)\rangle$ and need to be calculated only once. This system of linear equations must be solved for the unknown $\mathbf{a}_j$ of the PCE. The details of this procedure were published in [5] and its implementation issues addressed in a number of other references [6,7]. These equations can be assembled into a matrix of size $(P + 1).n \times (P + 1).n$ (n being the number of degrees of freedom).

4. Numerical example: longeron with two uncertain interfaces

This example represents an assembly of four plates including two uncertain interfaces on edges P5P6 and P7P8. A distributed moment of 1 Nm/m is applied on edge P5P8 and the geometric and mechanical properties are defined Fig. 2. The mean of the random variable associated with the two joints is the same ($\bar{k}_R = 5000$ Nm/rad), even though these two joints are independent.

The polynomial decomposition conducted using third-order polynomial functions leads to the computation of ten coefficients associated with ten Hermite's functions (Fig. 3). The graph (Fig. 4) representing the normalized value $\tilde{a}_1$ of the first element $a_1(\xi_1,\ \xi_2)$ at



Mechanical properties:

E = 2.1e11 Pa
n = 0.3
ρ = 7800 kg/m3
η = 0.1%
e = 8e-4 m

Points location (dimensions in meters):

p1	= 0.	0.	0.28
p2	= 0.3246	0.	0.28
p3	= 0.3246	0.05	0.28
p4	= 0.	0.05	0.28
p5	= 0.	0.	0.18
p6	= 0.2754	0.	0.18
p7	= 0.2754	0.05	0.18
p8	= 0.	0.05	0.18

Fig. 2. Characteristics of the longeron.

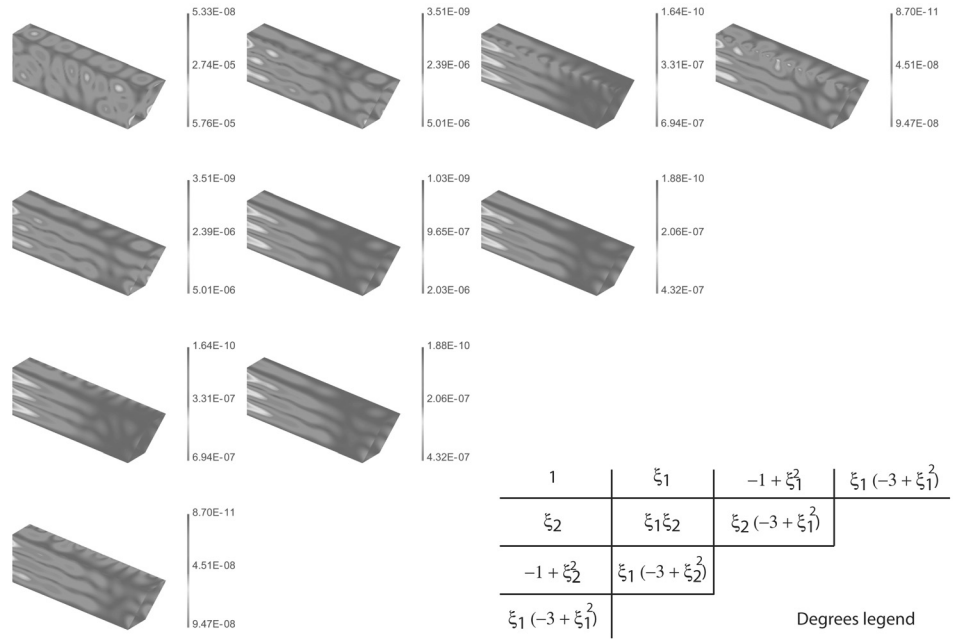


Fig. 3. The ten coefficients associated with the polynomial functions.

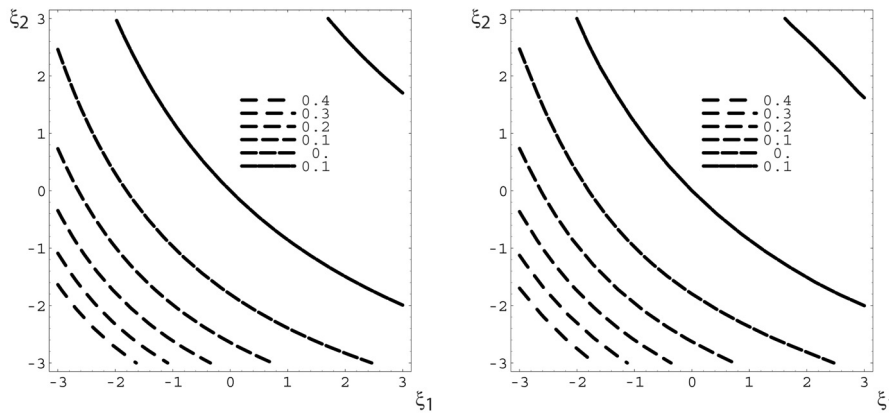


Fig. 4. Chaos representation (right) and exact parametric solution (left).

2200 Hz shows very good agreement between chaos and parametric solutions.

5. Conclusion

The Variational Theory of Complex Rays is a general approach to medium-frequency calculations. Its main features are the use of shape functions with a sound mechanical meaning and a dedicated variational formulation allowing the use of independent approximations in the different substructures. In some cases, such as assemblies of beams, the VTCR leads to the exact solution in tension as well as in bending. For complex structures, the VTCR seems to be a very promising theory due to its ability to give predictive results at a very low numerical cost. It can also take into account complex connections among substructures, described by stochastic parameters, in order to represent the physical reality at the lowest possible numerical cost. In this case, the benefit of two-scale shape functions is twofold: first, the global system defined by the PCE is relatively small; second, the smooth variation of the unknowns (generalized amplitudes) when the connection parameters vary can be easily represented by a polynomial function. Extensions of this approach to structural heterogeneities [8], transient dynamics and frequency bandwidth [9] are in progress.

References

[1] Lyon R, Maidanick G. Power flow between linearly coupled oscillators. JASA 1967;34(5):623–639.
[2] Ladevèze P, Arnaud L, Rouch P, Blanzé C. The variational theory of complex rays for the calculation of medium-frequency vibrations. Eng Comput 2001;18(1–2):193–214.
[3] Rouch P, Ladevèze P. The variational theory of complex rays: a predictive tool for medium-frequency vibrations. Comput Methods Appl Mech Eng 2003;192;3301–3315.
[4] Cameron R, Martin W. The orthogonal development of non linear functionals in series of Fourier-Hermite functionals. Ann Math 1947;48(16):385–392.
[5] Ghanem R, Spanos P. Stochastic Finite Elements: A Spectral Approach. Berlin: Springer, 1991.
[6] Ghanem R, Kruger R. Numerical solution of spectral stochastic finite element systems. Comput Methods Appl Mech Eng 1996;129:289–303.
[7] Ghanem R. Ingredients for a general purpose stochastic finite elements implementation. Comput Methods Appl Mech Eng 1999;168:19–34.
[8] Blanc L, Blanzé C, Ladevèze P, Rouch P. A multiscale and trefftz computational method for medium-frequency vibrations of assemblies of heterogeneous plates. Comput Assist Mech Eng Sci 2003;10:375–384.
[9] Ladevèze P, Rouch P, Riou H, Bohineust X. Analysis of medium-frequency vibrations in a frequency range. J Comput Acoustic 2003;11(2):255–283.