Length scales evolution and localization phenomenon in sand

Mustafa I. Alsaleh[a,*], George Voyiadjis[b], Khalid A. Alshibli[b]

[a]Machine Research, Caterpillar Inc., Mossville, IL 61552, USA
[b]Department of Civil & Environmental Engineering, Louisiana State University, Baton Rouge, LA 70803, USA

Abstract

A numerical model in the Cosserat continuum for strain localization phenomena in granular materials was developed by the authors. The mathematical formulations used in the present numerical model are equipped with evolution equations for the length scales through micropolar theory. The evolution equations of the internal length scales describe any possible change in the contact surface between the particles, damage to the particles if it exists and/or any change in the local void ratio within the domain. The solution for the shear bands thickness shows more accurate correlation with the experimental results and less dependency on the mesh size when such evolution equations are used.

Keywords: Strain localization; Granular materials; Evolution of length scales; Micropolar theory

1. Introduction

Granular materials exhibit a complex mechanical behavior when subjected to high plastic deformation. The microstructure of the particles and how they affect the material behavior is essentially justified by the non-uniformity in the shape and surface roughness at the micro or even at the nano level. During the hardening regime, granular materials behave as a continuum until the failure or instability point, where deformations begin to localize into a small but finite shear zone, called the shear band. Thicker shear bands indicate less shear strength; the angle of inclination of such bands with respect to the minimum principal axis gives an indication of the stability of the soil mass. Since the shear band properties are essentially dependent on some micro-length scale it is important to carefully incorporate this parameter.

The Cosserat theory can successfully separate the grain rotation from its translation adding three additional degrees of freedom to any point in the 3-D continuum. Granular materials undergo high rotational and translational deformations at failure. The classical strain tensor fails to capture the real kinematics such as micro-rotation in granular materials and other alternative tensors need to be introduced instead [1,2].

* Corresponding author. Tel.: + 1 309 578 3245; Fax: + 1 309 578 4277; E-mail: Alsaleh_Mustafa_I@cat.com

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The present model assumes two different length scales embedded in the Cosserat-based formulations. These length scales are adjusted using the effect of the shape indices and the surface roughness.

Evolution equations for the proposed length scales and their effects on the plastic strain localization are introduced in this paper. Such equations are then implemented into the numerical model to study their effect on the plastic strain localization in granular materials.

2. Proposed evolution equations for the length scales

The length scales are affected by the shape and the surface roughness of the particles; such effects are accounted for through the following equations:

$$l_s = \frac{I_{SPH}}{I_R} l_{ave} \tag{1}$$

where l_s, I_{SPH}, I_R and l_{ave} are the length of the surface of contact, sphericity index, roundness index and the mean particle size (equivalent to d_{50}), respectively.

This equation is proposed to account for the sphericity and roundness effect on the length of the contact surface. All researchers assume that the length of the contact surface is the mean particle size; however, it is believed that this quantity will be affected by the shape of the particles. The length of the contact surface increases as the sphericity index increases and decreases as the roundness index increases. Based on this argument Eq. (1) was proposed.

The length of the arm of rotation is assumed to have the following expression:

$$l_a = \frac{I_R}{I_{SPH}} l_{ave} + R_a \quad (2)$$

where l_a and R_a are the length of the arm of rotation and the mean surface roughness respectively. Similar to the length of the contact surface, the length of the arm of rotation is assumed to be affected by the shape of the particles. Equation (2) shows that this quantity increases as the roundness index and surface roughness increase and decreases as the sphericity index increases.

The above shape indices are defined as follows:

$$I_{SPH} = \left| \frac{D_{equ}}{d_s} - \frac{D_{equ}}{d_L} \right| \quad (3)$$

$$I_R = \frac{P_{act}}{\pi\left(\frac{d_s + d_L}{2}\right)} \quad (4)$$

in which P_{act} is the actual perimeter of the particle, D_{equ} is the sphere-equivalent diameter, and d_s and d_L are the shortest and longest dimensions, respectively [3].

In Eqs. (1) and (2) it is assumed that for a given element, which will involve few particles, all the parameters are held constant throughout the simulation and only the mean particle size l_{ave} will evolve based on the cumulative effective plastic strain and the point or Cosserat rotation. It is worth noting here that the particles are treated as rigid bodies (no particle damage); however, the evolution equations are used here to account for the changes in the suggested length scales due to the translational and rotational deformations of the nonuniform-shaped particles. Following Garcia [4] the following evolution equation is proposed to update the average length scale:

$$l_{Ave} = \frac{f \delta d_{50}}{\delta + d_{50} p^{\frac{1}{m}}} \quad (5)$$

where d_{50} is the mean particle size, m and f are material constants, δ is a constant taken as mean surface roughness of particles, and p is the effective plastic strain as follows:

$$p = \int_0^t \sqrt{\dot{\gamma}''_{ij} \dot{\gamma}''_{ij}} \quad (6)$$

A simpler evolution equation was used in this paper as

$$l_{ave} = l_o e^{-k_1 p} \quad (7)$$

where k_1 is a constant coefficient taken here as unity and l_o is the initial length scale, which is assumed to be as follows:

$$l_o = \frac{\frac{I_{SPH}}{I_R} d_{50} + \frac{I_R}{I_{SPH}} d_{50} + R}{2} \quad (8)$$

It is important to mention here that Eq. (7) has no physical basis especially for granular materials since it assumes an exponential decay for the initial length scale. The chosen length scales can decrease or increase based on the shape of particles and the final deformation in each loading step.

In this paper we propose a new evolution equation for the length of contact and the arm of rotation as a function of the shape indices and the Cosserat rotation. It is assumed here that such length scales are subjected to significant changes during plastic deformation attributed to the rotation of the particles. As the particle rotates and translates it causes the length scales to change because of the high non-uniformity in their shapes. Based on this argument the following equations are proposed and used in this paper:

$$l_s = \frac{I_{SPH}}{I_R} d_{50} \sin(\omega^c) \quad (9)$$

$$l_a = \frac{I_R}{I_{SPH}} d_{50} \cos(\omega^c) + R_a \quad (10)$$

The above equations are proposed based on a thorough understanding of the behavior of granular materials. As mentioned earlier, the particles will exhibit large translational and rotational deformations due to the discreteness of such materials. The rotation of the particles will cause the assigned length scales to change due to the high irregularity in the particle shape.

3. Results and discussion

The numerical model used in this paper is a Cosserat continuum-based model, which enables us to incorporate the discussed length scales. The model was used to predict the behavior of the very dense F-75 Ottawa sand. It was found that the numerical solution has some minor mesh dependency if one chooses to use a constant length scale, while it is not mesh-dependent if an evolution equation is used for the internal length scale. Figure 1 shows the difference between the predicted shear bands using a constant and evolving length scale (Eq. 10) for two different mesh sizes. In Fig. 1(a) the predicted shear band using a constant length scale was found to have some mesh dependency, while in Fig. 1(b) with a variable length scale it was found that mesh dependency decreases and consistent shear band thickness was

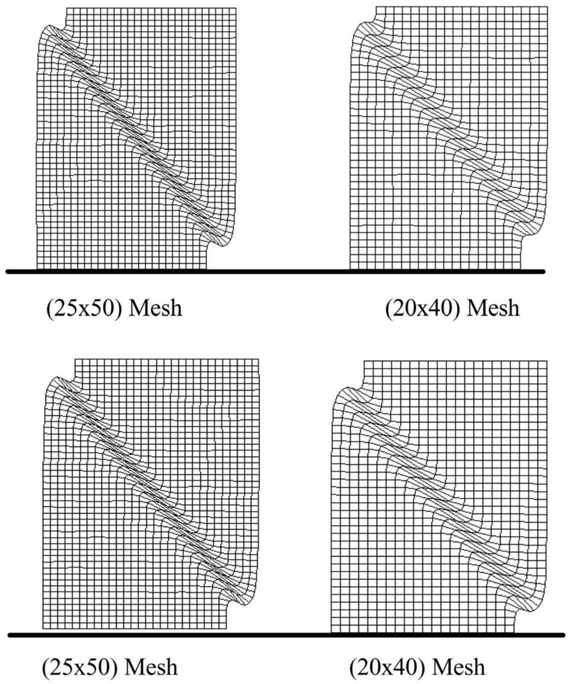

Fig. 1. Effect of the length scale and mesh size on shear band: (a) constant internal length scale; (b) variable internal length scale.

obtained using a different mesh size. Three different evolution Eqs. (6) through (10) were used and all gave similar solutions for the shear band thickness and inclination. However, comparing the predictions with experimental results in Alshibli [5], it was found that Eq. (10) gave closer predictions that compare better with the experiments for both the thickness and the inclination (see Tables 1 and 2). Figure 2 shows comparisons between model predictions and experimental results in

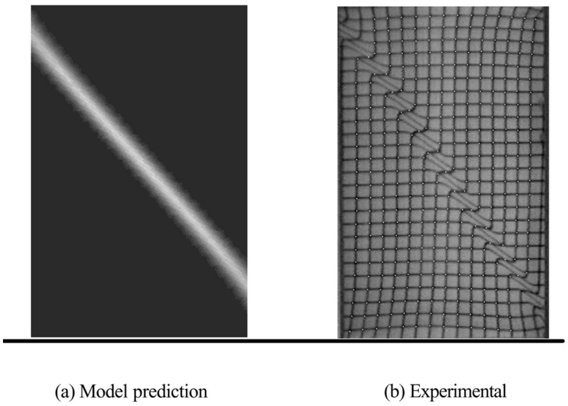

Fig. 2. Comparison between measured and predicted shear band thickness and inclination angle for the very dense F-75 Ottawa sand under confining pressure of 100.0 kPa. (a) Model prediction. (b) Experimental results [5].

Alshibli [5] under plane strain conditions for the very dense F-75 Ottawa sand. The numerical results compare well with the experimental observations.

4. Conclusions

The present numerical model was developed in order to accommodate any evolution expression for the internal length scales. Three different evolution equations were proposed and used in this paper; using these evolution equations was found to improve the numerical analysis for the strain localization in granular materials. The degree of mesh dependency was found to appreciably decrease once such evolution equations are used and the shear band thickness evolves similarly to the internal length scale during the deformation process.

Table 1
Comparison between measured and predicted shear band thickness

Material	Confining pressure (KPA)	Initial void ratio	Mean grain size (mm)	Measured shear band thickness (mm)	Predicted shear band thickness (mm)[a]	Predicted shear band thickness (mm)[b]	Predicted shear band thickness (mm)[c]
*F-sand	15.00	0.629	0.22	2.97	3.07	3.00	2.96
*F-sand	100.0	0.629	0.22	2.91	2.80	2.86	2.88
*F-sand	15.00	0.495	0.22	3.00	3.10	3.12	3.03
*F-sand	100.0	0.495	0.22	3.05	2.81	2.92	2.93
$^+$C-sand	15.00	0.767	1.60	17.33	17.96	17.90	17.52
$^+$C-Sand	100.0	0.767	1.60	17.00	16.61	16.74	16.92

* F-75 Ottawa sand, + Coarse silica sand
[a] Using Eq. (5)
[b] Using Eq. (7)
[c] Using Eqs. (9), (10)

Table 2
Comparison between measured and predicted shear band inclination angle

Material	Confining pressure (KPa)	Initial void ratio	Mean grain size (mm)	Measured shear band inclination (^{O})	Predicted shear band inclination (^{O})[a]	Predicted shear band inclination (^{O})[b]	Predicted shear band inclination (^{O})[c]
*F-sand	15.00	0.629	0.22	51.6	52.00	52.10	51.20
*F-sand	100.0	0.629	0.22	53.7	54.30	54.00	53.80
*F-sand	15.00	0.495	0.22	58.0	57.20	57.60	58.10
*F-sand	100.0	0.495	0.22	59.2	58.10	59.30	58.90
+C-sand	15.00	0.767	1.60	51.4	50.60	50.90	51.50
+C-sand	100.0	0.767	1.60	53.2	48.90	49.60	51.70

* F-75 Ottawa sand, + Coarse silica sand
[a] Using Eq. (5)
[b] Using Eq. (7)
[c] Using Eqs. (9), (10)

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