# Cylinder oscillations beneath a free-surface 

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#### Abstract

This investigation considers the wake states of a cylinder oscillating beneath a free-surface. The primary considerations are the variation of the forces on the cylinder and the structure of the near wake with the frequency of oscillation and the depth of submergence of the cylinder. The fully submerged oscillating cylinder exhibits two distinctly different wake states: low- and high-frequency. As the cylinder approaches the free-surface many of the characteristics of these wake states persist. Notably, for both the free-surface and fully submerged cases, the transition between the two states corresponds to a jump in the phase of the lift force, a shift in the phase of vortex shedding and a distinct change in the structure of the near wake. The mean vorticity fields show that as the cylinder approaches the free surface the changes in the oscillating cylinder wake are generally consistent with those observed for a stationary cylinder. Close to the free-surface both the low- and high-frequency wakes become non-symmetric, as does the stationary cylinder wake; and in all cases there is a net negative lift force on the cylinder.


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## 1. Introduction and discussion

The wake of a stationary cylinder in a free-stream has a natural self sustained instability resulting in the formation of the strongly periodic Kármán street, where the natural frequency of the stationary cylinder wake is designated $f_{o}$. The perturbation, amplification or elimination of the natural instability has been the subject of extensive and ongoing investigation. In this paper the combined effect of two mechanisms which can alter this natural instability, the presence of a free-surface and the forced large scale oscillation of the cylinder at frequencies close to $f_{o}$, are considered. The two individual cases of a stationary cylinder beneath a free-surface and a fully submerged oscillating cylinder have been extensively studied, however the combined effect of these mechanisms has received little, if any, attention.

### 1.1. Stationary cylinder beneath a free-surface

When the flow past a stationary cylinder is bounded by a free-surface both the structure of the near wake and the forces on the cylinder are altered. For these flows the two most important parameters are the normalised depth of the cylinder below the free-surface, $h / D$, and the Froude number $F r=U / \sqrt{g D}$. Previous studies [1,2] have found that as a cylinder approaches either a solid or free-surface the pressure distribution around the cylinder becomes increasingly non-symmetric and the cylinder experiences a net lift force away from the surface. Additionally, $[1,2]$ found that at very small values of $h / D$ the periodic fluctuation of the near wake was essentially eliminated, indicating that vortex shedding is suppressed when the cylinder is very

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Fig. 1. Stationary cylinder beneath a free-surface. Instantaneous vorticity fields showing the variation of wake mode with submergence depth $h / D$ : (a) Mode I, $h / D=1.0$; (b) Mode II, $h / D=0.125$; (c) Mode III, $h / D=0.079$, where for all cases $R e=2100$ and $F r=0.166$. The small dot in the corresponding lift traces represents the time at which the image was acquired.
close to a surface. The PIV measurements of Sheridan et al. [3] demonstrated that close to the free-surface the structure of the near wake exhibits a number of different states which are distinctly different from the fully submerged cylinder wake. As $h / D$ decreases Carberry et al. [4,5] observed three different wake states, which were described in terms of both the lift force on the cylinder and the structure of the near wake. These three wake states, termed mode I, II and III, are shown in Fig. 1 and are consistent with the wakes seen by others experimentally [3] and numerically [6]. The present study relates these modes to the fluctuating lift, as indicated by the direct comparison o the lift traces with the PIV images in Fig. 1. The mode I wake is essentially a modified Kármán wake: the mode of vortex shedding is very similar to the fully submerged case and, as shown by the force insert in Fig. 1(a), the lift force is highly periodic. However, compared to the fully submerged case shown in [5] the streamwise length of the attached vorticity is significantly shorter, and there is a corresponding increase in the amplitude of the fluctuating lift force. Closer to the free-surface the mode II and III wakes, shown in Fig. 1 (b) and (c) respectively, are clearly altered by the presence of the free-surface; the attached wakes are elongated and the lower shear layer has a distinct downwards angle. The close proximity of the free surface for the mode II and III wakes appears to suppress periodic vortex shedding and the lift traces in Fig. 1 (b) and (c) have only a small fluctuating component. The major difference between the mode II and III wakes is the behavior of the fluid flowing between the cylinder and the free-surface. For the mode II wake the flow over the top of the cylinder remains attached to the free-surface and, as shown by Sheridan et al. [3], at higher Froude numbers large scale free-surface waves form downstream of the cylinder. The mode III wake occurs at lower values of $h / D$ where the gap between the cylinder and the free-surface is smallest. The fluid moving over the top of the cylinder separates from the freesurface resulting in the formation of a jet. The jet consists of negative vorticity generated on the upper surface of the cylinder and positive vorticity generated as the free-surface deforms above the cylinder. At $F r=0.166$ the jet of fluid is very thin and
consequently is not well resolved in Fig. 1(c). However, at higher Fr the mode III wake occurs at deeper submergence depths and generates a wider jet, as shown in Fig. 3 of Sheridan et al. [3] at $F r=0.6$, where the mode III wake occurs at $h / D=0.31$ and 0.43 . After the jet has separated from the free-surface it can either remain attached to the cylinder or form a free jet angling downwards across the back of the cylinder.

### 1.2. Fully submerged oscillating cylinder

Controlled excitation of a cylinder at frequencies close to the natural Kármán frequency causes significant changes in both the structure of the wake and the forces on the cylinder. Bishop and Hassan [7] found a simultaneous jump in the phase and amplitude of the lift force as the frequency of forced transverse oscillation, $f_{e}$, passes through $f_{o}$. This jump in the lift force has been observed by a number of investigators over a wide range of oscillation amplitudes, $A / D$, and Reynolds numbers. Fig. 2 contains data from a number of experiments, [8-12] showing the phase, $\phi_{L}$, and amplitude, $C_{L}$ of the lift force as a function of $f_{e} / f_{o}$. The jump in the lift force around $f_{e} / f_{o}=1$ appears to be a generic feature of these wakes, although as discussed by [12,5] the nature of the jump of the lift force can vary with both $A / D$ and $\operatorname{Re}$. Changes in the structure of the near-wake as $f_{e} / f_{o}$ passes through unity have been observed by a number of investigators, [5,12-14]. These changes have been described either as a change in the mode of vortex shedding or as a switch in the timing of the phase referenced initial vortex. Simultaneous measurement of the near wake flow field and the forces on the cylinder by [12] conclusively showed that the jump in $\phi_{L}$ and $C_{L}$ corresponds to a transition between two distinctly different wake states. The wake states on either side of the transition were found to have characteristic features which persist over a wide range of $f_{e} / f_{o}$. For values of $f_{e} / f_{o}$ below the transition the low-frequency state is characterised by a lift force which is small in amplitude and approximately out-of-phase with the displacement of the cylinder. The low-frequency wake has long extended shear layers and at $A / D=0.5$ the mode of vortex shedding is clearly 2 P , with two counter rotating pairs shed per oscillation cycle. Above the transition the lift force for the high-frequency state is much larger in amplitude and is close to being in-phase with the cylinder's motion. The high-frequency wake is clearly different from the low-frequency wake: two single vortices are shed per oscillation cycle (2S) and the length of the attached wake in both the streamwise and transverse direction is shorter. The evolution of the low- and high-frequency wakes is discussed in detail by [12]. For the low- and high-frequency wakes the phase point in the oscillation cycle at which large scale vortex shedding occurs is clearly different. Thus, at the transition between the two wake states the jump in the lift force corresponds to a large shift in the phase of vortex shedding.


Fig. 2. Data from a number of previous investigations showing the variation of (a) the lift phase and (b) the lift amplitude with $f_{e} / f_{o}$. For all cases $A / D=0.5$ while $R e$ ranged between 2300 and 60000 .

### 1.3. Current investigation

As described above, previous investigations of a stationary cylinder beneath a free-surface and a fully submerged oscillating cylinder have identified a number of characteristic features which vary with either $f_{e} / f_{o}$ or $h / D$. These results raise a number of interesting questions for the more complicated case of a cylinder oscillating beneath a free-surface:

Are the features which characterised the effect of the free-surface on the flow over a stationary cylinder, i.e., the downwards angle of the wake and the net negative lift force on the cylinder, present for the oscillating case?

As the depth of the cylinder beneath the free-surface varies are there systematic changes in the wake structure and lift forces which are analogous to the three wake modes of the stationary cylinder?

As the frequency of oscillation passes through $f_{e} / f_{o}=1$ is there a transition between two distinctly different wake states which is analogous to the transition between the low- and high-frequency states of the fully submerged oscillating cylinder?

## 2. Experimental method

The experiments were performed at Lehigh University in a free surface water channel with working section of width 914 mm , depth 609 mm and length 4928 mm . The free stream velocity, $U$ was $0.090 \mathrm{~m} \cdot \mathrm{~s}^{-1}$ with a turbulence level less then $0.1 \%$. A high resolution stepper motor was used to oscillate the cylinder such that its vertical motion transverse to the free-stream is given by:

$$
\begin{equation*}
y(t)=A \sin \left(2 \pi f_{e} t\right) \tag{1}
\end{equation*}
$$

In all cases $A / D$ was equal to 0.5 , while during each set of experiments the frequency of oscillation varied over the range $0.76<f_{e} f_{o}<1.05$. The Froude number, $\operatorname{Fr}=U / \sqrt{g D}$, was 0.166 , which is the same value as for the stationary cylinder presented in Fig. 1. The depth of the cylinder below the free-surface, $h$, is measured at the uppermost point in the oscillation cycle of the cylinder and three different cylinder submergence depths were considered: $h / D=3 / 16,3 / 8$ and 10 (the fully submerged case where the effect of the free-surface is considered to be negligible).

The span averaged vertical force on the cylinder was measured using strain gauges configured in a full Wheatstone bridge. The lift force was calculated by subtracting the time varying inertia force, due to the acceleration of the cylinder's mass, from the measured force trace. For the range of flow and oscillation parameters considered the lift force was "locked-on" to the motion of the cylinder and the time varying lift force on the cylinder can be approximated by a sinusoidal function:

$$
\begin{equation*}
C_{L}(t)=C_{L} \sin \left(2 \pi f_{e} t+\phi_{L}\right) \tag{2}
\end{equation*}
$$

where $C_{L}$ is the amplitude of the lift coefficient and $\phi_{L}$ is the phase of the lift force with respect to the cylinder's displacement $y(t)$. Both $C_{L}$ and $\phi_{L}$ were calculated in the time domain using data points corresponding to more than 400 cylinder oscillations. The flow field was measured using laser scanning PIV as described in detail by [15]. The images were recorded on high resolution 35 mm film and digitised at 106 pixels $/ \mathrm{mm}$. The interrogation of the images was performed using a window size of $90 \times 90$ pixels with an overlap ratio of 0.50 , resulting in velocity fields with $3500-3700$ vectors. The mean vorticity fields presented in this paper were calculated by averaging eight instantaneous vorticity fields per oscillation cycle over 2-4 oscillation cycles.


Fig. 3. Schematic showing the experimental system.


Fig. 4. Variation of lift phase (degrees) with $f_{e} / f_{o}$ for three different submergence depths.

## 3. Results and discussion

In this section results for a cylinder oscillating at $h / D=3 / 16$ and $3 / 8$ are compared with the results for the fully submerged oscillating cylinder. The variation of the lift phase with $f_{e} / f_{o}$ for the free-surface cases, shown in Fig. 4 , is surprisingly similar to the fully submerged case. As the frequency of oscillation increases the characteristic jump in $\phi_{L}$ is observed at both $h / D=3 / 8$ and $3 / 16$. In both cases the value of $f_{e} / f_{o}$ at which the jump occurs and the values of $\phi_{L}$ either side of the jump are very similar to the fully submerged case. This indicates that close to the free-surface the wakes either side of the jump can be described in terms of low- and high-frequency states, where these states are directly analogous to those observed for the fully submerged cylinder. However, these wake states are altered by the presence of the free-surface and the nature of these changes will be discussed later in the paper. At $h / D=3 / 16$, and to a lesser extent at $h / D=3 / 8$, a number of intermediate values of $\phi_{L}$ occur during the jump in lift phase. This suggests that the nature of the transition between the low- and high-frequency states is affected by the presence of the free-surface.

Fig. 5 shows the mean vorticity fields for the low- and high-frequency states either side of the jump in $\phi_{L}$ for all three depths. The mean vorticity fields, rather than instantaneous fields, are shown because these better highlight the systematic changes due to the presence of the free-surface. The mean vorticity fields for the fully submerged low- and high-frequency states, shown in Fig. 5 (a(i)) and (b(i)) respectively, are remarkably similar to the mean fields obtained by [16] for a fully submerged elastically mounted cylinder. For the fully submerged cylinder both the low- and high-frequency wakes are symmetric about the horizontal axis and the net lift force on the cylinder is zero.

Fig. 5 shows that at $h / D=3 / 16$ and $3 / 8$ the mean vorticity fields in Fig. 5 are significantly modified by the presence of the free-surface and the wakes are clearly non-symmetric. Despite the changes which occur as $h / D$ decreases, in all cases the low-frequency wakes in Fig. 5(a) are distinctly different from the corresponding high-frequency wakes in Fig. 5(b). Moreover, examination of the instantaneous fields shows that as the depth of the cylinder is varied the phase of large scale vortex shedding does not change significantly. Therefore, for both the fully submerged and free-surface cases the transition between the two wake states corresponds to a large shift in the phase of vortex shedding. The presence of the free-surface appears to alter the formation of large scale vortex structures, with the vortex structures in the near wake becoming less coherent, both the mean and instantaneous fields are noisier.

For the symmetric fully submerged low-frequency wake, shown in Fig. 5 (a(i)), the majority of the negative mean vorticity is above the centre-line of the wake and the majority of the mean positive vorticity below the centre-line. The small lobes of oppositely signed vorticity close to the wake centreline are due to the formation of long shear-layers which, as described by [12], extend well across the back of the cylinder. However, the mean vorticity from these extended shear-layers is relatively localized and does not extend far downstream.

The low-frequency wakes in Fig. 5 (a(ii)) and (a(iii)) have an unbroken stream of negative mean vorticity starting at the upper surface of the cylinder and angling downwards across the centre-line of the wake. This indicates that over the oscillation cycle the negative vorticity dominates the corresponding positive vorticity angling upwards. As $h / D$ decreases from $3 / 8$ to $3 / 16$ the stream of negative vorticity becomes weaker but is at a steeper angle. This is consistent with the behavior of the upper shear layer for the stationary cylinder as $h / D$ decreases. Further downstream there is a large region of positive vorticity, evident in Fig. 5 (a(ii)) and (a(iii)). The vertical position of this positive vorticity is similar to the small "bubble" of positive vorticity in Figure $5(\mathrm{a}(\mathrm{i})$ ). However, in contrast to the fully submerged case, as the cylinder approaches the free-surface the region of


Fig. 5. Mean vorticity fields for (a) the low-frequency state and (b) the high-frequency state either side of the jump in $\phi_{L}$. In each case the fields are shown at 3 submergence depths: (i) fully submerged, (ii) $h / D=3 / 8$ and (iii) $h / D=3 / 16$. The lighter central cylinder represents the mean potion of the cylinder, while the two darker images represent the extreme positions of the cylinder corresponding to $A / D=0.5$.
mean positive vorticity extends well downstream and, particularly at $h / D=3 / 16$, the wake further downstream is dominated by positive vorticity.

The variation in the structure of wake for the high-frequency state as the nominal position of the cylinder is changed from fully submerged to $h / D=3 / 8$, and then to $h / D=3 / 16$, is quite complicated. The mean vorticity field of the fully submerged high-frequency wake is similar to the mean field for the Kármán wake of a stationary cylinder, with positive vorticity in the lower half of the wake and negative vorticity in the upper wake. The high-frequency wake at $h / D=3 / 8$, shown in Fig. 5 (b(ii)) is significantly distorted by the presence of the free-surface. A portion of the mean negative vorticity crosses the wake centreline but the region of negative vorticity is much smaller than for the corresponding low-frequency wake. Also, the vorticity wraps tightly around the base of the cylinder. The majority of the negative mean vorticity follows a path from the centre-line of the wake up to, and then along underneath, the free-surface. Examination of the instantaneous vorticity fields show that this region of negative mean vorticity corresponds to the movement of a relatively small but concentrated vortex structure along this path. At $h / D=3 / 16$ there is a more diffuse distribution of negative mean vorticity in the upper half of the wake and there is no stream of negative vorticity crossing the wake centre-line near the base of the cylinder. The instantaneous vorticity fields at $h / D=3 / 16$ show that a negative vortex structure moves up towards the free-surface but this structure is smaller than at
$h / D=3 / 8$. At other points in the oscillation cycle smaller less coherent negative vortex structures move downstream in the upper wake. In the mean vorticity field these two different regions of instantaneous negative vorticity combine to form the broad band of negative mean vorticity.

The structure of the high-frequency wakes close to the free-surface, shown in Fig. 5 (b(ii)) and (b(iii)), appear quite different. However, the changes in the mean wakes as $h / D$ decreases are consistent with a decrease in the strength of the negative vorticity crossing the centre-line of the wake and the formation of less coherent vortex structures. In these respects the variation of the high-frequency wake as $h / D$ decreases from $3 / 8$ to $3 / 16$ is similar to the variation seen in the low-frequency wake. At $h / D=3 / 8$ and $3 / 16$ the non-symmetric low- and high-frequency wakes share a number of features which can be attributed to the presence of the free-surface. In all cases the lower shear layer has a downwards angle away from the free-surface; this was also observed for the stationary cylinder. Additionally, as the oscillating cylinder is brought closer to the free-surface the non-symmetry of the wake structure, including the generation of net positive vorticity, results in a net negative lift force on the cylinder. These observations are consistent with the net time-averaged downwards force on a cylinder oscillating in the streamwise direction beneath a free-surface characterised by Cetiner and Rockwell, [17].

## 4. Conclusions

This paper describes the wake states of a cylinder that is oscillating underneath a free-surface. The presence of the freesurface and the large scale forced oscillations are both mechanisms that act to alter the wake's natural Kármán instability. This investigation considers the case where these mechanisms are combined; the results are then related to the two individual cases of a stationary cylinder below the free-surface and a fully submerged oscillating cylinder. Many of the features which characterise the effect of the free-surface on the stationary cylinder wake are also present for the oscillating cases. As $h / D$ decreases both the stationary and oscillating cylinder wakes have a downwards angle away from the free-surface and there is a net negative lift force on the cylinder. At small submergence depths the natural instability of the stationary cylinder wake is altered to the extent that periodic vortex shedding is not observed. It is difficult to directly compare the depth at which periodic vortex shedding ceased for the stationary cylinder with the oscillating case, as during the oscillation cycle the cylinder's depth varies. However, as $h / D$ decreases the close proximity of the free-surface appears to inhibit, but not eliminate, periodic vortex shedding from the oscillating cylinder. For a given wake state, as $h / D$ decreased the vortex structures shed into the wake were less well defined and the vorticity was more widely distributed throughout the wake.

As $f_{e} / f_{o}$ increases, the jump in $\phi_{L}$ that characterises the transition between the low- and high-frequency states for a fully submerged cylinder was also observed at $h / D=3 / 8$ and $3 / 16$. At $h / D=3 / 8$ and $3 / 16$ the variation of the forces on the cylinder and the wake structure with $f_{e} / f_{o}$ show that two wake states exist which are directly analogous to the low- and highfrequency states of the fully submerged cylinder. In all cases the phase of large scale vortex shedding and the value of $\phi_{L}$ does not vary significantly with $h / D$ but depends primarily on $f_{e} / f_{o}$. However, the structure of the near wake, and possibly also the nature of the transition between the two wake states, was altered by the close proximity of the free-surface. For the flow and oscillation parameters considered here however, the vortex shedding appears to be primarily controlled by the motion of the cylinder.

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